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THE MULTIFACETED NATURE OF MATHEMATICS KNOWLEDGE
FOR TEACHING:

Understanding the Use of Teachers' Specialized Content Knowledge

and

the Role of Teachers' Beliefs from a Practice-based Perspective

by

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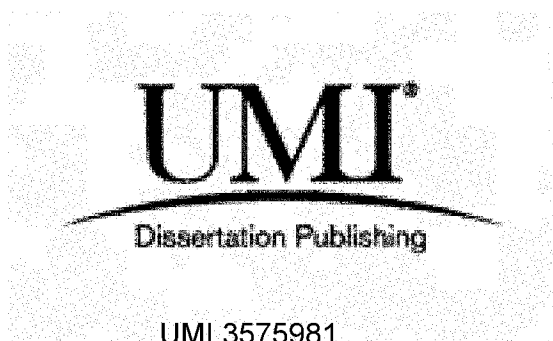
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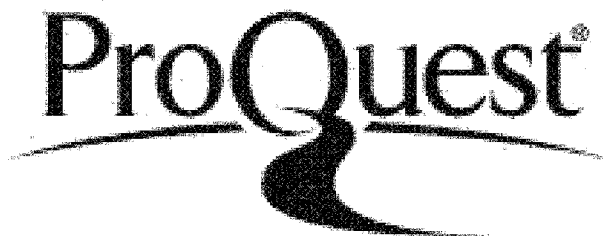


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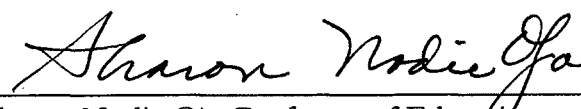
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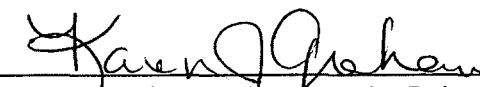
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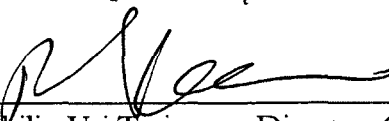
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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iii
LIST OF TABLES.....	ix
LIST OF FIGURES.....	xi
ABSTRACT	xii

CHAPTER	PAGE
I. CHAPTER 1: INTRODUCTION.....	1
II. CHAPTER 2: CONCEPTUAL FRAMEWORK.....	9
Introduction.....	9
I. Understanding SCK and the Organization of MKT.....	11
The Learning Mathematics for Teaching Research Group and the MKT Model.....	15
The Emergence of Specialized Content Knowledge (SCK).....	18
The Development of the Middle School Level MKT Survey: Measuring SCK.....	19
What is There Still to Learn About the Content of SCK in Middle School Level?.....	24
II: Understanding SCK and the Role of Teachers' Beliefs in Practice.....	25
Pedagogical Content Beliefs in the Content Area of Mathematics.....	28
An Illustration.....	32
What is There Still to Learn About the Manifestation of Beliefs within SCK Use?.....	33
Conclusion.....	35
III. CHAPTER 3: METHODOLOGY	
The MKT Survey and Dataset.....	37

Data.....	38
Participants.....	39
Distributing the Middle School MKT Survey and Gathering Results.....	40
Missing Data.....	41
A History of the Development of the Measures of CCK and SCK.....	44
Expanding MKT Work to Assess Middle School MKT.....	44
Reliability and Validity.....	45
Setting the Stage for Analysis.....	47
Summary.....	55
Part II: Understanding How Teacher Beliefs are Manifested in Use of SCK.....	56
Introduction.....	57
A Pragmatic Approach to the Extension of a Previous Study.....	58
Choosing a Participant Pool.....	59
Section A: MKT Survey.....	60
Section B: Beliefs Survey.....	61
Section C: The Structured Interview.....	67
Why MKT Tasks and the Think-Aloud Approaches Were Chosen.....	69
Section D: The Focused Interview.....	70
Section E: Observations.....	71
Reliability and Validity.....	74
Summary.....	75
IV: CHAPTER 4: DATA ANALYSIS AND FINDINGS	77
Descriptive Statistics.....	79
Testing the Fit Model 1.....	84

Testing the Fit for Model 2.....	95
Testing the Fit for Model 3.....	101
Cross-validation.....	105
Summary.....	107
Understanding How Teachers' Beliefs are Manifested in SCK Use.....	113
Methods Overview.....	115
Study Participants.....	119
Preliminary Analyses: Five Data Sources.....	120
Analysis of MKT Survey Data.....	120
Analyzing the Use of Specialized Content Knowledge.....	132
Additional Analyses.....	134
Results of Analysis.....	137
Inconsistency in the Beliefs-to-Practice Connection: Surface Beliefs.....	138
Background and Setting.....	140
Beliefs About the Nature of Mathematics.....	143
Beliefs About Mathematics Learning.....	144
Beliefs About Mathematics Teaching.....	145
Robert's Use of Specialized Content Knowledge in Practice.....	146
Julie's Use of Specialized Content Knowledge in Practice.....	150
Summary.....	157
Consistency in the Beliefs-to-Practice Connection: Limited SCK Use.....	158
Background and Setting.....	159
Beliefs About the Nature of Mathematics.....	163
Beliefs About Mathematics Learning.....	165

Beliefs About Mathematics Teaching.....	145
Debra's Use of Specialized Content Knowledge in Practice.....	166
Nancy's Use of Specialized Content Knowledge in Practice.....	171
Summary.....	176
Consistency in the Beliefs-to-Practice Connection: Opportunities for SCK Use.....	176
Background and Setting.....	177
Beliefs About the Nature of Mathematics.....	181
Beliefs About Mathematics Learning.....	181
Beliefs About Mathematics Teaching.....	182
Sarah's Use of Specialized Content Knowledge in Practice.....	183
Summary.....	188
Consistency in the Beliefs-to-Practice Connection: Mixed Beliefs and an Illustration of Teaching Quality.....	189
Background and Setting.....	189
Beliefs About the Nature of Mathematics.....	195
Beliefs About Mathematics Teaching and Learning.....	196
Jake's Use of Specialized Content Knowledge in Practice.....	198
Anna's Use of SCK in Practice.....	204
Summary.....	207
V. CHAPTER 5 DISCUSSION AND INTERPRETATIONS.....	210
Specialized Content Knowledge as the Learner's Building Blocks.....	212
The Development and Nature of Beliefs.....	214
The Role of Context.....	216
Dichotomies.....	217

Emphasis of Conceptual Knowledge Versus Skills/Practice.....	217
Socio-constructivist Versus Traditional Beliefs.....	218
Other Dichotomies.....	219
The Role of Teacher Education.....	219
Study Limitations.....	220
Future Questions and Challenges.....	221
LIST OF REFERENCES.....	222
APPENDIX A: SURVEYS AND INTERVIEW GUIDES.....	233
APPENDIX B: IRB PAPERWORK.....	234

LIST OF TABLES

1. Beswick's (2005) Relationships between Beliefs.....	30
2. The Organization of MKT Survey Tasks for Model 1.....	48
3. Belief Survey Subscales (Staub & Stern, 2002).....	62
4. An Expansion of Beswick's (2005) Relationships between Beliefs.....	64
5. Raymond's (1992) Criteria for the Categorization for Teachers' Beliefs about Mathematics Teaching.....	64
6. Ball's (2005) Mathematical Tasks of Teaching.....	71
7. Raymond's (1992) Criteria for the Categorization of Teachers' Mathematical Teaching Practice.....	71
8. Teacher Characteristics.....	79
9. Teacher's Educational Background (%).....	80
10. Organization of Tasks Model 1.....	86
11. Model 1: Goodness of Fit Statistics.....	87
12. Organization of Tasks Model 2.....	95
13. Model 1, 2: Goodness of Fit Statistics.....	97
14. Residuals.....	99
15. Model 1, 2, 3: Goodness of Fit Statistics: Sample 1.....	102
16. Model 1, 2, 3: Goodness of Fit Statistics: Sample 2.....	105
17. Ball's (2005) Mathematical Tasks of Teaching.....	117
18. An Expansion of Beswick's (2005) Relationships between Beliefs.....	125
19. Raymond's (1992) Criteria for the Categorization for Teachers' Beliefs about	

Mathematics Teaching.....	126
20. Selected Indications of Nancy's Belief System.....	129
21. Summary of the Beliefs about Teaching Mathematics Cluster Categorization.....	130
22. Ball's (2005) Mathematical Tasks of Teaching.....	131
23. Sample Descriptions of SCK Use.....	133
24. Raymond's (1992) Criteria for the Categorization of Teachers' Mathematical Teaching Practice.....	134
25. Participant Background Information.....	210

LIST OF FIGURES

1. A Subset of the Proposed MKT Model (LMT Research Group, 2013).....	42
2. The Proposed Model 1.....	49
3. The Proposed Model 2.....	52
4. The Proposed Model 3.....	53
5. Model 1 Overview.....	82
6. Model 2.....	96
7. Model 3.....	101
8. Overview of the Data Collection Methods in Part II of the Study.....	113
9. Above, All Case Study Participants, Ordered by IRT Scores on Middle School Patterns, Functions, and Algebra Section of the MKT Survey, Ranging from -3 to 3.....	120
10. Above, All Case Study Participants, Ordered by IRT Scores on Middle School Numbers and Operations Section of the MKT Survey, Ranging from -3 to 3.....	121
11. Above, All Case Study Participants, Ordered by IRT Scores on Elementary Patterns, Functions, and Algebra Section of the MKT Survey, Ranging from -3 to 3.....	121
12. Overview of Themes from Analysis.....	136
13. Julie's Shape Discussion.....	150
14. Anna's Shape Discussion.....	205
15. Overview of Results.....	208

ABSTRACT

THE MULTIFACETED NATURE OF MATHEMATICS KNOWLEDGE FOR
TEACHING:
UNDERSTANDING THE USE OF TEACHERS' SPECIALIZED CONTENT
KNOWLEDGE AND THE ROLE OF TEACHERS' BELIEFS
FROM A PRACTICE-BASED PERSPECTIVE

by

Lauren E. Provost

University of New Hampshire, September, 2013

This work investigates middle school teachers' mathematics knowledge for teaching (MKT) as defined by Hill (2007). Within this two-part dissertation, the level of MKT was considered as well as the role of teacher beliefs in actual specialized content knowledge (SCK) use, a specific type of mathematics knowledge for teaching vital in quality mathematics instruction. Additionally, the model of MKT knowledge was explored through confirmatory factor analysis on a large, national dataset of middle school mathematics teacher survey responses involving mathematics knowledge for teaching. SCK was found to be vital in quality mathematics instruction yet not sufficient. Teacher beliefs about the delivery of mathematics instruction ultimately acted as a filter, at times limiting SCK use, even if a teacher held high levels of SCK. The mathematics knowledge that teachers hold is highly complex; confirmatory factor analysis results indicated that we have yet to truly capture the essence of MKT, yet the importance of understanding such knowledge is clearly essential. Implications for preparing future teachers are discussed.

CHAPTER 1

THE IMPORTANCE OF SPECIALIZED CONTENT KNOWLEDGE

First, what we are doing in this country is unethical. We let people start teaching who have not yet demonstrated that they can perform. And, further, the students who most need skillful and highly effective teachers are least likely to get them. Second, we know how to change this and must do so deliberately and without delay.¹

Over the last twenty years, what constitutes quality mathematics instruction has drawn increased attention from stakeholders in mathematics education. Policymakers are particularly concerned with the mathematics knowledge teachers hold. This is due in part to accumulating evidence that students learn more when they are taught by more mathematically knowledgeable teachers (Hill, Rowan & Ball, 2005; Rowan, et al., 2001). In fact, quality mathematics teaching necessitates an array of mathematics knowledge typically interwoven within instructional characteristics such as providing classroom environments that are rich in accurate mathematics language, encourage connections between topics and concept generalizations, provide student-accessible explanations, and offer multiple problem solving procedures or representations (Hill, et al., 2005; National Council of Teachers of Mathematics (NCTM, 2000). However, Hill (2007) found that mathematically knowledgeable teachers are not the norm in classrooms across the United States. In fact, mathematics teachers often enter and remain in mathematics teaching without the mathematics knowledge and skills for teaching necessary to deliver quality mathematics

¹ Excerpt from Dr. Deborah Loewenberg Ball's Summary of Testimony to the U.S. House of Representatives Committee on Education and Labor on May 4, 2010.

instruction (e.g., Ball, 1990; Battista, 1999; Ma, 1999; NCTM, 1989, 1991, 2000; NRC, 1989). “Allowing teachers to learn at children’s expense is unethical. We must build a system for ensuring that new teachers have the requisite professional skills and know how to use them” (Ball & Forzani, 2010).

The drive for improvement in teachers’ mathematics knowledge stems from multiple sources. Our nation is significantly underperforming in mathematics as compared to other nations leading to valid concerns about our future economic status as a country (National Commission on Teaching and America’s Future, 1996; National Center for Education Statistics, 2007; U.S. Department of Education, 2010). Within our nation, the achievement gap in mathematics still exists (NCES, 2011). The lack of an equitable distribution of quality mathematics teachers is concerning and not unique to students predominantly within high-poverty and high-minority areas. However, students within high-poverty and high-minority areas are even more likely to experience *less* mathematically knowledgeable teachers (Balfanz & Byrnes, 2006; Hill, 2007). There is widespread agreement among stakeholders in mathematics education that understanding and improving teachers’ mathematics knowledge is imperative in addressing these concerns (NCTM, 2000; U.S. Department of Education, 2008).

Research aimed at understanding and improving teachers’ mathematical knowledge is not new. In the past, mathematics teacher knowledge has been typically defined as a composite of a teachers’ mathematics courses taken after Calculus, the number of math methods courses taken, and a major or minor in mathematics, typical of process-product research of the sixties and seventies (Mullens, Mernane & Willett, 1996; Rowan, Chiang & Miller, 1997). Imprecise definitions of teachers’ mathematical content knowledge such as these resulted in little progress in understanding teachers’ mathematics knowledge, as well as

a misspecification of the causal processes linking mathematics teachers' knowledge to students' learning (Rice, 2003).

In 1986, Lee Shulman and his colleagues developed a conceptual framework that encompassed *pedagogical content knowledge*; not only the knowledge a teacher has accrued (i.e., the wisdom of practice) but how this knowledge is used in classrooms (Shulman, 1986; Wilson et al., 1987). Shulman was not the first to investigate pedagogical content knowledge, although his contributions have sparked decades of research in mathematics content knowledge use and continue to be referenced to date (Carpenter, Franke, and Levi, 2003; Grossman, 1990; Wilson & Wineburg, 1988). Although Shulman's work was substantial, the mathematical knowledge used in the practice of teaching had yet to be fully understood and conceptualized, particularly as it relates to the actual practice of mathematics teaching. Continuing this endeavor, Ball, Thames, and Phelps built upon Shulman's work, developing a practice-based theory of mathematics knowledge used in teaching.

"The mathematical content teachers must know in order to teach has yet to be mapped precisely...past methods lack the power to propose and test hypotheses regarding the organization, composition and characteristics of content knowledge [core content and pedagogical] for teaching" (2008, p. 43).

Researchers in mathematics education agree that in order to further the understanding of teachers' mathematical knowledge use, a more direct approach is necessary (Blanton & Kaput, 2005).

Deborah Ball and the Learning Mathematics for Teaching (LMT) research group at the University of Michigan have taken a direct, practice-based approach in understanding and creating measures of mathematics teacher knowledge. In doing so, the LMT research team has created a practice-based model representative of the types of knowledge and skills

mathematics teachers hold, which they refer to as the Mathematics Knowledge for Teaching (MKT) model (Ball, 2008).

A survey was developed by the LMT research group (i.e., the MKT Survey), incorporating tasks that reflect the use of this knowledge *as used in the practice of teaching*. MKT survey tasks represent two main subdomains of teacher knowledge: subject matter knowledge and pedagogical content knowledge. This work has led to significant understanding of mathematics knowledge used in teaching mathematics as well as a robust survey tool used to further understand the role of MKT in professional development, teacher preparation, student achievement and other teacher quality issues with policy implications (Ball, 1990; Ball, Goffney & Bass, 2005; Ball & Hill, 2004; Hill, Ball & Schilling, 2008; Hill, Rowan & Ball, 2005).

Most notably, past research of the LMT research group have uncovered a special type of knowledge used by teachers, which the LMT research group has named *specialized content knowledge* (SCK). This type of mathematics teacher knowledge, SCK, is knowledge crucial to quality mathematics instruction. Teachers use this specific type of decompressed mathematical knowledge to look for patterns in student errors, examine student solutions to see if the solution will work in other similar situations, and provide an explanation as to why a particular student solution works or not. Each of these activities represents mathematical knowledge unique to the profession of mathematics teaching and critical in the delivery of quality mathematics instruction.

Most of the initial MKT survey tasks, however, were written specifically for teachers' mathematical knowledge applicable to *grades three through six*. More work is necessary to understand specialized content knowledge at the middle school level, a key period of time in

a student's development of algebraic ideas and a major indicator of later success in mathematics.

“Fewer efforts have focused on teachers' knowledge of student thinking about algebraic ideas in middle grades – a period that marks a significant transition from the concrete arithmetic reasoning of elementary school mathematics to the increasingly complex, algebraic reasoning required for high school mathematics and beyond” (Alibali et al., 2007, p. 251).

Further justifying the need for understanding MKT at the middle school mathematics level is that students' mathematical performance sharply declines in middle school years, where for most students, Algebra begins (RAND Mathematics Study Panel, 2003; U.S. Department of Education, 2008). Middle school years and Algebra in particular, serve as gateway knowledge to later success in high school and eventually college-level mathematical courses and widen career choices (NCTM, 1989; 2000). Hence, expanding the understanding of teachers' specialized content knowledge at the middle school level is critical.

In light of these concerns, I have chosen to situate my study within middle school mathematics teaching, typically encompassing Algebraic concepts. The research questions guiding this study are:

1. What specialized content knowledge do middle school mathematics teachers hold? In particular, how does this knowledge differ from common content knowledge
2. How are pedagogical content beliefs, an additional component of teacher knowledge, manifested in specialized content knowledge use

In the first part of my study, I will investigate what constitutes middle school mathematics teachers' specialized content knowledge and describe such knowledge, focusing on specific mathematical topics that play a prominent role in the middle school curriculum: numbers and operations, patterns, functions, equations and inequalities (Hill, 2007; Nathan & Koedinger, 2000; Sfard, 1995). Specifically, I will use confirmatory factor analysis techniques to understand the content and organization of both common and specialized

content knowledge through the analysis of MKT survey responses resulting from a large, nationally representative sample of middle school teachers. The dataset resulted from a 2005-2006 administration of the MKT survey tasks, as reported by Heather Hill in her 2007 article, entitled *Mathematical Knowledge of Middle School Teachers: Implications for the No Child Left Behind Policy Initiative*.

In Part II of my dissertation, I take a closer look at specialized content knowledge. Specialized content knowledge, although necessary for quality teaching, is only one type of knowledge necessary for quality mathematics teaching. Another form of teachers' knowledge, teacher beliefs, has been regarded for decades as having a significant influence on mathematics teaching (Anderson & Piazza, 1996; Battista, 1994; Ernest, 1991; Yadav & Koehler, 2004). For example, teachers who believe students construct their own knowledge as a result of instruction generally make significantly different choices in their uses of mathematical knowledge, representations and other pedagogical tools impacting student learning in considerable ways (Staub & Stern, 2002). For example, Peterson, Fennema, Carpenter, and Loef (1989) found that first grade teachers with a constructivist perspective more often used specific specialized counting strategies to teach numeracy as an approach to solving word problems, resulting in significantly greater increases in numeracy word problem-solving achievement than students of teachers of a less constructively-based perspective. In another compelling example, Raymond (1997) found that instructional practices in the math classroom are more influenced by beliefs about specialized content knowledge than by beliefs about mathematical pedagogy.

Studying specialized content knowledge along with teachers' pedagogical beliefs specifically from a practice-based, content-specific perspective has been suggested to provide a more comprehensive picture of how mathematics content knowledge is used in teaching

(Peterson, et al., 1989; Raymond, 1997). There are very few such studies, however. The MKT tasks developed by the LMT team provide ideal practice-based tasks in which to investigate the use of MKT and teacher beliefs (Ball, Thames & Phelps, 2008). Hence, after gathering a greater understanding of the content and organization of specialized content knowledge teachers hold resulting from Part I of my study, I then use middle school MKT survey tasks classified as *specialized content knowledge* in multiple ways to understand the manifestation of beliefs in SCK use. Multiple sources of evidence will be gathered from ten middle school mathematics teachers in multiple state, including: (1) participant results of the middle school MKT survey, (2) survey results of teachers' pedagogical content knowledge beliefs written to coincide with specialized content knowledge use, (3) a structured interview, where participants will reason through specialized content knowledge tasks via a think-aloud interview protocol, (4) a focused interview where participants view their own first interview by video and provide retrospective comments, and (5) observational data in the classroom for a minimum of three full-class periods, while participants are engaging in middle school mathematics lessons. Multiple sources of data were strongly suggested due to the complexity of how teachers' beliefs ultimately unfold in the use of specialized content knowledge (SCK), as well as the ongoing struggles to uncover and expand upon the understanding SCK use in practice (Beswick, 2008; Handal, 2003).

“Knowledge is important, but alone it is not enough to account for the differences between mathematics teachers” (Ernest, 1989, p. 1). Teachers' beliefs also impact almost every aspect of instruction, including specialized content knowledge use (Sowder, et al., 1998). Hence, in conclusion, a primary goal of this study is to provide a rich and cohesive picture of how specialized content knowledge is used in the classroom, both furthering the understanding of middle school teachers' SCK and of the role of teachers' beliefs in the use

of this knowledge. Only by furthering research in both areas will we have a more holistic understanding of how specialized content knowledge is used, addressing the initial concerns of preparing quality mathematics teachers in the future.

CHAPTER 2

CONCEPTUAL FRAMEWORK

The purpose of this study is to further the understanding of two key types of knowledge middle school teachers hold: middle school teachers' specialized content knowledge (SCK) and pedagogical content knowledge beliefs, as well as the interaction between the two in practice. The first part of Chapter 2 consists of a discussion of how the existing body of research on mathematics teachers' specialized content knowledge currently attempts to explain the content and organization of this knowledge and the role of this knowledge in quality mathematics teaching, framing the first research question: *What specialized content knowledge do middle school mathematics teachers hold? In particular, how does this knowledge differ from common content knowledge?*

I begin with a discussion of how current research that attempts to explain what constitutes SCK as it presents itself in quality mathematics teaching and its relationship to student learning. Before I proceed, it is necessary to note what is meant by *quality mathematics teaching* as defined formally by Hill and her colleagues at the University of Michigan and the Harvard Graduate School of Education (2010), particularly how it is defined in relation to the *Mathematical Quality of Instruction* (MQI) observational instrument. The MQI instrument, in particular, was written to identify important dimensions of quality classroom mathematics teaching shown to have a significant impact on student learning (Ernest, 1989; Charalambous, 2006; Hill, et. al, 2008; Hill, et. al, 2010; Kersting, 2008; Sowder, et al. 1998; Stigler, 2009; Swafford, et al., 1997) and includes the following characteristics: the richness of the mathematics, student participation in mathematical

reasoning and meaning-making, and the clarity and correctness of the mathematics covered in class, among others.

The chapter discussion will then move to a description of the work done by the LMT Research group at the University of Michigan, in creating a practice-based model of mathematics knowledge in teaching. The discussion will include all the domains and topics in the Mathematics Knowledge for Teaching Model (MKT Model); however, the focus of this study is on a specific type of knowledge mathematics teachers hold, *specialized content knowledge*. One of the most significant findings in research involving the MKT Model was that specialized content knowledge is a special type of knowledge specific to the profession of mathematics teaching, yet different than mathematics knowledge non-teachers hold. For example, a teacher who can understand and teach procedures involving operations on fractions may not have the specialized knowledge to explain the conceptual basis for the operations or the knowledge that allows for communicating, such knowledge to third-grade students (Hill, 2007).

In the second part of Chapter 2, I discuss the conceptual framework behind my second research question: *How are middle school algebra teachers' pedagogical content beliefs manifested in specialized content knowledge use?* I consider this additional critical component of teachers' knowledge heavily influencing mathematics teaching; teachers' pedagogical content knowledge beliefs. There is a general consensus among researchers in mathematics education that mathematics teachers' pedagogical content knowledge beliefs play a major role in guiding their classroom behavior and therefore influence student learning (Clark & Peterson, 1986; Ernest, 1989; Thompson, 1992; Fang, 1996; Wilson & Cooney, 2002). I provide an overview of research in this area, focusing on understanding the relationship between mathematics teachers' specialized content knowledge and the role of teachers'

pedagogical beliefs in the use of this knowledge. In particular, I point out a lack of research base in this highly complex area.

Lastly, each of the two parts of Chapter 2 conclude with some additional limitations of current research in each of the two areas and how the current study addresses these limitations. Both sections intend to emphasize the need to understand both teachers' specialized content knowledge and pedagogical content knowledge beliefs as well as the interplay between both, underlining the importance of both in providing a holistic picture quality mathematics teaching.

I: Understanding SCK and the Organization of MKT

Although there is little disagreement among stakeholders in education that quality mathematics teaching relies significantly on the mathematics knowledge teachers hold, research in this area over the past 25 years demonstrates a continuing struggle to define and conceptualize such knowledge. Interest in teachers' content knowledge dates back to the 1960s. Process-product research, also known as the educational production function literature of the sixties and seventies, was a beginning attempt at understanding and potentially measuring teachers' mathematical knowledge, as well as establish a relationship between this knowledge and student learning (Ball, 1990). During this time, teachers' mathematics knowledge was typically defined and sometimes measured as a composite of the number of math courses teachers had taken after Calculus, the number of math methods courses taken, and a major or minor in mathematics. Begle (1979), while working as a professor within both the mathematics and education departments at Stanford, conducted a meta-analysis of studies between the sixties to the mid-seventies that focused on investigating the relationship between mathematics teacher knowledge and student learning; a review that encompassed much of the process-product research of this time. The results

were inconsistent. Begle's results concluded that the number of math courses taken was positively associated with student achievement in only 10% of the studies reviewed, calling into question the role of teachers' pure subject matter knowledge in student learning.

The process-product research reviewed by Begle (1979) did not measure mathematics teachers' knowledge directly, a practice that by and large led to imprecise definitions of teachers' knowledge and misspecification of the causal processes linking teachers' knowledge to students' learning (Rice 2003). Rice also reviewed a wide array of studies examining mathematics teacher characteristics and the impact on student learning during a substantial period of time after Begle. As with Begle's 1979 review, Rice's 2003 review resulted in an array of literature that looked at teacher degree level, level of coursework, and other proxy measures of teacher knowledge with mixed results. In Rice's review, findings indicated that teacher coursework in both mathematics and pedagogy resulted in increases in student performance, yet this result was not consistent across grade levels. Rice also found that *methods courses*, i.e., those courses that taught a combination of content and pedagogy, consistently contributed to teacher effectiveness at all grade levels included in her review.

Dr. Lee Shulman also began a long quest for understanding the role of content knowledge and its use in teaching, but from a different perspective, beginning in the eighties. Shulman's interest in teacher knowledge was related to his deep commitment to teaching as professional work. Shulman specifically referred to teaching as a profession and as with other professions, teaching requires a specialized knowledge base (Shulman, 1987, 2000, 2005). Shulman's work in this area has been primarily conceptual and quite substantial, as he has been cited frequently over the years and his work led to the development of a framework for the National Board for Professional Teaching Standards (Shulman, 1987, 2000, 2005).

Shulman's (1987) work drew initially from the work of John Dewey, in the essay *The Child and the Curriculum* written in 1902, who differentiated between knowledge of the scientist (referred to by Dewey as logical understanding) versus knowledge for teaching (psychological understanding, according to Dewey [2009]). Under this assumption, Shulman introduced the idea of *pedagogical content knowledge (PCK)*. He explains that this type of knowledge (PCK) goes *beyond* core content knowledge and can be thought of as the "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (Shulman, 1987). PCK can be thought of as more than knowledge of a discipline and its facets; PCK is knowledge used *in the practice of teaching*. PCK consists of the most useful forms of representations, analogies, examples and explanations most useful for learning the key topics in mathematics. Shulman states, "...in a word, [PCK represents] the ways of representing and formulating the subject that make it comprehensive to others" (Shulman, 1986, p. 9). Shulman's definition of PCK also includes the misconceptions students are likely to have, how different representations are interpreted by students and how backgrounds of knowledge students bring with them influence new learning experiences (Shulman, 1986).

Although Shulman's ideas captured the essence of the type of knowledge researchers were trying to understand, researchers have used Shulman's framework to define the term PCK in varying ways over the years. For example, Leinhardt and Smith (1985) provided a conceptual framework for teacher knowledge, focusing on lesson structure knowledge and subject matter knowledge. The categories were further divided depending on different knowledge types of the expert and novice teacher. Also relying on Shulman's work, Nathan and Petrosino (2003) found that 48 pre-service secondary mathematics teachers differed in their understandings and judgments of student difficulties depending on the teachers' level

of mathematical training. That is, teachers with more advanced training in mathematics had views of student performance on algebraic reasoning that differed significantly from actual student performance patterns. Despite ongoing work in the area of teacher knowledge, the concept of teacher knowledge remained a roughly defined yet an important area of research.

The Knowledge of Mathematics for Teaching Algebra Project (Ferrinini-Mundy, et al, 2003) involves the creation of a conceptual framework for teachers' algebraic knowledge as well as the development of KAT (Knowledge of algebra for Teaching), an assessment of teachers' algebraic knowledge. This work has also furthered the understanding of the mathematics knowledge used in teaching in specific areas of algebra, yet is limited, however, by its focus on core algebra topics without the necessary key background concepts used in remediation during the teaching of algebra such as number sense (including fractions and other essential Algebra tools). Middle school teachers, including those teachers teaching algebra, spend a significant amount of time remediating in these areas (see Sleeman et al., 1989; Taylor & Francis, 1990); thus it is essential to understand such knowledge and its use.

Mathematics knowledge for teaching has been investigated in these many different ways over the last several decades. Charalambous (2006) completed an exhaustive review of such studies, including work involving the middle school teacher population. I now state the findings particular to the middle school teacher population:

- Teacher knowledge that is primarily superficial and procedural substantially constrains a teacher from providing a classroom environment with the dimensions of quality mathematics teaching (Borko et al., 1992; Eisenhart, Borko, Underhill, Jones et al., 1993; Sowder et al., 1998, Stein, Baxter and Leinhardt, 1990; Swenson, 1998).
- Teachers' strong mathematical knowledge can provide necessary and exceptional support in providing a classroom environment with the dimensions of quality teaching (Ball, 1992; Charalambous, 2006; Lloyd & Wilson, 1998; Lubinski, 1993).

Also along these lines, Swenson (1998) assessed four middle school teachers' mathematical knowledge and classroom teaching through pre-observation interviews followed by video-taped lessons of middle school content knowledge items. For these particular middle school teachers, they lacked explicit and connected knowledge, held traditional views about teaching and learning of mathematics (teaching by telling), lacked the important "big ideas" necessary for the content covered and held little understanding of potential misconceptions students hold, generally lacking the knowledge and skills needed to provide a classroom environment consistent with quality mathematics teaching.

The Learning Mathematics for Teaching Research Group and the MKT Model

In 2008, after reviewing the above inquiries involving mathematics knowledge for teaching, Hill, Schilling and Ball offered:

"Despite this wealth of research, we argue that the mathematical content teachers must know in order to teach has yet to be mapped precisely...past methods lack the power to propose and test hypotheses regarding the organization, composition and characteristics of content knowledge [core content and pedagogical] for teaching" (p. 18).

Deborah Loewenberg Ball and her colleagues at the University of Michigan and beyond, the Learning Mathematics for Teaching research team, have been investigating the idea of mathematics knowledge in teaching (MKT) for over a decade using a more direct measure of teacher knowledge; knowledge as captured in the practice of teaching.

The LMT research team's direct approach stems from their concerns that current theories on mathematics teachers' knowledge were conceptually based versus practice-based (Ball, 1990). The LMT research group presents an effective argument that understanding and conceptualizing this knowledge would be best accomplished by first looking at the practice of quality mathematics instruction (Ball, Schilling & Hill, 2004). In fact, their conceptualization of mathematics knowledge rooted in quality instruction is a central and

unique aspect of their work, likely an aspect that has resulted in years of success in furthering the knowledge of MKT.

Initial interest in the project was driven by a variety of reasons, including a 1988 study by Ball who developed interview questions that revealed significant inadequacies of in-service and pre-service teachers' knowledge of important mathematical topics necessary for quality teaching (Ball, 1990). Additionally, Ball (1990, p. 252) found in a large, university-based study of 252 preservice teachers, that teachers "brought with them to teacher education from their precollege and college mathematics experiences, understandings that tended to be rule-bound and thin." One might argue that this would be resolved by teachers taking more mathematics courses. However, the participants from this study were secondary teachers having majored in mathematics with substantial coursework in mathematics prior to entering the teacher education program (Ball, 1990). Also, elementary teachers with a greater number of math classes in the same study were found to have similar issues in their lack of conceptual understandings in the same topic areas as the elementary teachers having taken fewer classes in mathematics (Ball, 1990).

Highly concerned with these matters, Ball continued her interest in this area in the following years with her work and the work of her colleagues culminating in the development of the MKT model and the MKT survey tool, discussed in the research article *Content Knowledge for Teaching: What Makes it Special?* (Ball, Thames & Phelps, 2008). The MKT model and its survey tool are based upon a practice-based conceptualization of mathematics teachers' knowledge and includes two main domains, Subject Matter Knowledge (otherwise known as content knowledge) and Pedagogical Content Knowledge.

Each domain includes teacher knowledge vital to quality teaching. Core common knowledge, for example, "is the mathematical knowledge and skill used in settings other than

teaching” (Ball, Thames & Phelps, 2008). Teachers need to be able to do basic computations and comparisons with numbers. These skills are not necessarily distinct from other educated adults or even mathematicians. Although a vital component of teacher knowledge, this type of knowledge is not sufficient for quality teaching. Specialized content knowledge, the focus of this study to be described in more detail below, is the mathematical knowledge and skills unique to teaching.

It is important to note, particularly in relation to the first part of this study, that specialized content knowledge is closely tied to, and sometimes undifferentiable, from common content knowledge. An example discussion given by Ball, Thames and Phelps (2008) involving a teacher choosing a numerical example useful in investigating a student’s understanding of decimal numbers demonstrates the distinction between CCK and SCK. Teachers must first be able to choose and order the decimal numbers (CCK), then choose the list in such a way that it will bring the key concepts to the forefront necessary for learning (SCK).

Ball includes three other categories of mathematical teacher knowledge beyond CCK and SCK, building on Shulman’s earlier work. Knowledge of content and students (KCS) encompasses the intersection between teacher’s knowledge of mathematical ideas and how students come to understand these ideas. Student common misconceptions and mathematical thinking would be housed here. Misconceptions students typically have, such as confusion with parentheses, require different representations and explanations in teaching. The domain Knowledge of Content and Teaching (KCT) encompasses knowledge used in choosing examples, sequencing instruction, choosing instructional formats, and finding representations most likely to be accessible to a certain grade level. It is important to stress the multi-faceted interrelatedness of each knowledge category.

The Emergence of Specialized Content Knowledge (SCK)

Around the year 2000, the LMT research group continued the conceptualization of MKT and began the development of survey measures of MKT. A distinctive domain of subject matter knowledge surfaced during the LMT's work; *specialized content knowledge*. Specialized content knowledge for teaching (SCK) is described by Ball, Phelps & Thames (2008, p. 40) as "the mathematical knowledge and skill unique to teaching." That is, teachers use a type of decompressed mathematical knowledge (SCK) to look for patterns in student errors, examine student solutions to see if the solution will work in other similar situations, and provide an explanation as to why a particular student solution works or not.

The identification and description of Specialized Content Knowledge was a major advance in the understanding of mathematics teachers' knowledge necessary for quality instruction. A particularly noteworthy finding is that specialized knowledge for teaching mathematics at the elementary school level exists independently from Common Content Knowledge (CCK). Ball et al. (2005) confirmed this finding empirically; that is, the MKT survey developed was used in finding that MKT SCK survey items were statistically separable from MKT CCK items through the use of confirmatory factor analysis, among other related analyses (Ball et al., 2005; Hill, 2007).

SCK is clearly vital for quality mathematics teaching. Teachers need more than common content knowledge. Teachers must also possess mathematical knowledge that "goes beyond what is needed to carry out an algorithm reliably" (Ball et al., 2005, p. 22). Teachers spend most of their time "interpreting someone else's error, representing ideas in multiple forms, developing alternative explanations and choosing a usable definition" (Ball, 2003, p. 8). "Teaching quality might not relate so much to performance on standardized tests on mathematics achievement as it does to whether teachers' knowledge is procedural or

conceptual, whether it is connected to big ideas or isolate into small bits, or whether it is compressed or conceptually unpacked” as in the form of SCK (Hill & Ball, 2004, p. 332). Mathematics education researchers agree that SCK, or lack thereof, strongly affects teaching quality and ultimately student achievement (Ball & Wilson, 1990; Graeber, 1999; Hill, 2007; Lee et al., 2003).

In fact, teachers with SCK knowledge and skills can develop unpacked knowledge of mathematical knowledge (multiple representations of core ideas, different interpretations of mathematical operations), develop detailed knowledge of classroom mathematical practices (using mathematical language, providing rich mathematical experiences for students), explain student thinking and move students’ forward in obtaining understanding based upon the students’ own thinking, proving, posing questions, explaining representing (Ma, 1999).

To understand CCK and SCK, and the differentiation between the two, which relates to my first research question, I chose the technique of confirmatory factor analysis and related analyses to attempt to clarify the distinction between CCK and SCK. In order to proceed to analysis of Part I of my study, it was necessary to review MKT work done specifically at the middle school level in understanding CCK, SCK and the differentiation between the two.

The Development of the Middle School Level MKT Survey: Measuring SCK

Building upon the work done by the LMT research team at the elementary school level, Hill (2007) developed the middle school MKT survey. In doing so, Hill relied upon the work that was done at the elementary level as a foundation. Hence, I first discuss the relevant work that was done in building the elementary level MKT survey. The initial MKT item-writing team included mathematics education researchers, psychometricians, teachers and other professionals, pulling from their own extensive experience in mathematics

education and most importantly, teaching, to create the MKT survey questions based on essential mathematical topics (Ball, Hill & Schilling, 2004). The goal was to capture the essence of teacher knowledge in quality teaching, basing much of their work on previous research shown to correlate with student learning (Abell, 2008; Blanton & Kaput, 2005; Carpenter, Fennema, & Franke, 1996; Rasmussen & Marrongelle, 2006). As Heather Hill described in an October 2010 MKT workshop hosted at the Harvard Graduate School of Education:

“We watched hours and hours of thousands and thousands of teachers teaching from across the country. These were videotapes that multiple teams of researchers in California and Michigan have accumulated over 10 years, teachers with a diverse set of backgrounds in a diverse set of schools and school populations. These videos captured teachable moments in mathematics classrooms that expert teachers wait a lifetime or more to be exposed to” (Personal communication, November 3, 2010).

There were initially 138 items developed for the MKT survey which were further subdivided into four topic types (number concepts, operations, and patterns, functions and Algebra). Although it was not initially clear how items were categorized, through extensive exploratory and confirmatory factor analysis and a significant of time rewriting, the items factored into the domains and topics shown in Figure X on page 17, the hypothesized MKT model (Ball, Phelps & Thames, 2008).

The MKT surveys were initially piloted in California’s Mathematics Professional Development Institutes including 40 sites serving 23,000 K-12 teachers. The sample participants were paid for their work over several week-long professional development sessions. Three forms of the test were administered: 640 participants took form A, 535 took form B and 377 took form C. Items on all three forms tended to perform consistently across the three forms in the factor analysis with minor exceptions (Hill, Ball & Schilling, 2004).

The LMT research group also addressed the issues of reliability and validity throughout this time. First, the LMT research group obtained access to the teachers in the above sample in a format more suitable for understanding the validity of the MKT content knowledge; the classroom (Hill, et al., 2008). Additionally, teachers participated in structured think-aloud MKT task sessions to work through each of the survey tasks to verify that survey task knowledge did in fact reflect knowledge teachers use in the classroom. These interviews led to significant insights into teacher thinking and the use of MKT (Ball, 1990). After this was completed, the surveys were reevaluated with a team of psychometricians to rectify any outstanding issues with reliability and validity. Since 2001, the LMT research group has continued its work with psychometricians, practicing teachers, and mathematics educators from across the country to expand and revise MKT survey items, reevaluating the reliability and validity of the survey measures. Multiple nationally representative samples of teachers in multiple contexts have been used to ensure reliability and validity (Hill, et al., 2008).

The best argument for the use of the elementary level MKT survey as a partial foundation for the middle school MKT survey, however, comes from how it has been used to uncover consistent findings involving understandings of teacher knowledge, teacher education, and student learning. For example, Heather Hill and her colleagues found that teachers' MKT did relate to increases in students' performance after controlling for key student and teacher-covariates (including student socioeconomic status, teacher's credentials and experience) through the use of the MKT survey tool on a large national representative sample of both students and teachers (Hill et. al, 2005). Students taught by teachers in the top MKT quartile gained two weeks of instruction compared to their counterparts taught by teachers with average MKT scores (Hill et. al, 2005). Additionally, the effect of teacher

knowledge was investigated in relation to students' socioeconomic status, finding that "while teachers' mathematical knowledge would not by itself overcome the existing achievement gap, it could prevent the gap from growing" (Hill, et al., 2005, p. 44).

Additionally, Hill and Ball (2004) found gains in MKT for teachers who participated in professional development programs focused on mathematics teaching methods. Specifically, the gains were larger for teachers who participated in programs that focused more on proof, analysis and use of representations than other programs available. Current studies are under way to replicate these above findings (Geoffrey Phelps, LMT Research Team, personal communication, November 14, 2010). The MKT survey continues to be used to demonstrate a relationship among MKT, quality instruction (MQI) and student learning based at multiple research sites via independent research groups (Hill, et. al, 2008; Hill; Umland, Litke & Kapitula, 2010).

Of the items previously written from the elementary level MKT survey, two concepts were chosen for the construction of the middle school survey: (1) numbers and operations and (2) patterns, functions and algebra. Math educators, mathematicians, professional developers, the research project group and current and former teachers constructed additional middle school survey items (Hill, 2007). The majority of the item-writers were the same professionals who wrote the initial elementary level MKT survey items. Keeping in line with the elementary school work, item writers drew on their extensive experience in and knowledge of teaching mathematics, cutting edge research on mathematics knowledge used in teaching, and observing classroom instruction while writing and reviewing items over a year's period of time (Hill, 2007).

The middle school MKT survey items fell into two categories: numbers and operations, and patterns, functions and algebra. These two categories were chosen for

several reasons. An estimate from the Third International FIX (according to Hill, 2007) shows that 40% of eighth grade lessons in the United States focus on numbers and operations (Peak, 1996). Concepts included in Patterns, Functions and Algebra also play a critical role in the middle school years (Hackenburg, 2005; Stephens, 2007; NCTM, 2010). Hence, the 2005 MKT Survey resulted in 92 items and fell approximately evenly between the two concepts of numbers and operations and patterns, functions and algebra, in specific areas such as: whole, rational and integer numbers and operations, ratio, proportion and percent, radicals, linear, quadratic and exponential functions, Algebraic expressions and equations, absolute value and inequalities (Hill, 2007).

As discussed earlier, the delineation between Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK) is not always clear. When Hill (2007) administered the middle school MKT Survey on a smaller subset of a demographically representative population of approximately 1,000 middle school teachers in the United States, scales created to represent the SCK and CCK theoretical constructs “were correlated at .79, where .81 would be a perfect correlation, accounting for measurement error. These strong correlations, along with the factor analysis results, suggest a one-factor model” (p. 73). Despite these initial findings of a one-factor model, there is compelling research to suggest otherwise (Hill, Dean & Goffney, 2006; Hill & Ball, 2004; Hill et al., 2006; Hill, Rowan & Ball, 2005). In my study, I will use techniques similar to Hill’s (2007) in attempts to understand and clarify the organization of CCK and SCK. In particular, I will use confirmatory factor analysis and related techniques to differentiate between CCK and SCK, if possible, within the current MKT model. I will reflect upon these results and the overall conceptualization of the MKT model in attempts to revise the original model to clarify the organization of MKT, specifically the important CCK and SCK distinction.

What Is There Still to Learn about the Content and Organization SCK at the Middle School Level?

Despite the extensive work of the LMT research group and others researching in the area of mathematics knowledge use in teaching, there is further work to be done to *fully* uncover and understand the multi-faceted and complex specialized content knowledge effective mathematics teachers hold. In particular, the first notable limitation is that most of the capturing of the specialized knowledge has previously focused on the elementary school level. Understanding teachers' specialized content knowledge at the middle school level, and in particular of Algebra, is a critical. Hence, in the first part of my study, I focus specifically on understanding teachers' MKT use of topics that are key in the middle school curriculum, including topics central in the study of Algebra.

An additional need is to further verify the organization of the hypothesized MKT model with a focus on the subdomains SCK and CCK. The difference has been established empirically at the elementary school level (Hill, Dean & Goffney, 2006; Hill & Ball, 2004; Hill et al., 2006; Hill, Rowan & Ball, 2005). The delineation of the two constructs SCK and CCK is not as clear at the middle school level (Hill, 2007). Hence, in my study I will use datasets gathered by Hill (2007) to readdress the hypothesized delineation between SCK and CCK through similar confirmatory factor analysis (and related techniques) used by Hill (2007). I will use these results to inform the current MKT model, with respect to understanding CCK, SCK and the differentiation between the two constructs.

Specialized content knowledge is not used in isolation and is more often than not enmeshed with other key knowledge that influences teaching. One of the most significant forms of knowledge influencing teaching practice is teacher pedagogical content beliefs. In order to obtain a greater understanding of SCK use, I investigate SCK in the practice of

teaching with the consideration of this major teacher filter in decision-making: teachers' pedagogical content beliefs. I now turn to framing my second research question: *How are pedagogical content beliefs, an additional component of teacher knowledge, manifested in specialized content knowledge use?*

II: Understanding Specialized Content Knowledge and the Role of Teachers'

Beliefs in Practice

Specialized content knowledge is not used in isolation and is more often than not enmeshed with other key types of knowledge that influence teaching, such as teachers' pedagogical content beliefs. In fact, teachers' pedagogical content beliefs have been regarded for decades as critical to the reform of mathematics teaching practices (Anderson & Piazza, 1996; Battista, 1994; Cooney & Shealy, 1997; Ernest, 1989). Specifically, teacher pedagogical content beliefs are thought to play a substantial role in *how* content knowledge is used in the classroom (Beswick, 2008; Buehl & Fives, 2009). However, few studies have investigated teacher beliefs along with the use of mathematics content knowledge, including SCK, due to the highly complex nature of both. Hence, in Part II of my study, I have chosen to use qualitative methods to explore and explain teachers' beliefs as teachers use specialized content knowledge in the classroom. I begin by discussing past and current research in the area of pedagogical content beliefs.

Despite limited understanding of how teacher pedagogical content beliefs (PCK beliefs) actually influence instruction and varying results in research on teacher beliefs, there remains a general consensus that PCK beliefs play a major role in guiding their classroom behavior and therefore influence student learning (Clark & Peterson, 1986; Ernest, 1989; Fang, 1996; Thompson, 1992; Wilson & Cooney, 2002). Past research demonstrates that we do not fully understand the role of teachers' beliefs within the complex context of the

classroom. To begin, there is no agreed upon definition of *beliefs* across research in this area (McLeod & McLeod, 2002). Most broadly, the term *belief* is defined in research on teacher education as “a psychologically held understanding, premises or proposition about the world that is felt to be true” (Richardson, 1996).

Although some conceptualizations of teachers’ beliefs treat beliefs and knowledge as being entirely different, there are several conceptualizations that provide a strong argument for teachers’ beliefs as teachers’ knowledge (Ernest, 1989; Leatham, 2006). As Leatham (2006, p. 7) states, having done extensive research in the area of mathematics teachers’ beliefs:

Of all the things we believe, there are some things that we “just believe” and other things we “more than believe – we know.” Those things we “more than believe” we refer to as knowledge and those things we “just believe” we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as subsets of the things we believe.

The classification of teachers’ beliefs as knowledge is one of the many complexities in this area of research. An additional controversy involves the nature of the link between teacher beliefs and practice, with some authors reporting consistency between teacher beliefs and practice (e.g., Stipek, Givvin, Salmon & MacGyvers, 2001; Thompson, 1984) and others finding inconsistent relationships between the two (Cooney, 1985; Shield, 1999). Most agree, however, that the beliefs-to-practice connection can be clarified in a consistent manner, if contextual factors influencing the enactment of beliefs are taken into consideration. According to Handal (1995), some constraints in the school system such as the school community, school administration, or classroom environment, may be reasons why inconsistencies occur between teacher beliefs and their actual teaching practice.

The heart of the inconsistencies ultimately fall on the actual definition of teacher beliefs and teacher belief systems. In the seventies, Green (1971) created a conceptual framework

for teachers' beliefs, a framework that continues to be used effectively in understanding teachers' beliefs across disciplines to date (Beswick, 2005; Handal, 1995; Mcleod, 1998). According to Green (1971), teachers hold *systems* of beliefs that are highly complex. This is because teacher beliefs are not held in isolation from each other but are inter-related in complicated ways. Further complicating the understanding of such systems, teachers may or may not be conscious of the beliefs they hold. Also, individual beliefs may take on a variety of forms; a belief may be fact or opinion or an attitude that is manifested as a belief, ultimately taking the form of knowledge in when a belief is put into action (Liljedahl, 2008).

Green (1971) identifies three dimensions of teacher beliefs systems that continue to be vital in investigating teacher beliefs:

- There is a logical relation between beliefs (beliefs are primary or a derivative of other beliefs).
- Relations between beliefs are influenced by the strength of beliefs (central beliefs are strongly held, peripheral beliefs are less strongly held in relation to central beliefs).
- Beliefs are organized in clusters (a cluster of beliefs can be held in isolation from other clusters).

Green's first property of beliefs can be illustrated as follows. Consider a teacher who believes that constructivism *is* an important philosophy of teaching to hold. This same teacher might also believe that cooperative learning is necessary for the successful application of a constructivist teaching philosophy. For this particular teacher, there is a logical relationship between these beliefs. That is, if a teacher believes in this philosophy, there is likely a set of beliefs that logically follow from this belief.

Green's second property of beliefs involves the centrality of beliefs. Centrally held beliefs are strongly rooted and, not surprisingly, difficult to change. Centrally held beliefs have consequential beliefs, beliefs related to the central belief within the belief system but may be held peripherally. In contrast with centrally held beliefs are beliefs that are peripherally held, also called surface beliefs (Kaplan, 1991). Surface beliefs are not rooted in the teachers' central belief system. Rather, teachers hold such beliefs because they think they should or to conform to some expectation. Agreeing with this idea, Hoyles (1992) explains that all beliefs are situated; even the construction a belief or a belief system is highly influenced by social context.

Lastly, beliefs can cluster; a group of beliefs may be held in isolation from another cluster of beliefs. A teacher might have a specific cluster of beliefs about how to teach the concept of balancing equations, which can be distinct from another belief cluster about how to teach properties of polygons. Beswick (2005, p. 41) notes that an important consequence of Green's concept of clusters is that "inconsistencies between beliefs held in different clusters are likely to go unnoticed and hence unreconciled."

Pedagogical Content Beliefs in the Content Area of Mathematics

Ernest (1989), among others (McLeod, 1998; Thompson, 1991) have identified three general categories of beliefs specific to the teaching and learning of mathematics, as well as the subject itself, based on Green's earlier work (1971). Ernest (1989, p. 1) states, "Knowledge is important, but alone it is not enough to account for the differences between mathematics teachers. Two teachers can have similar knowledge, but while one teaches mathematics with a problem solving orientation, the other has a more didactic approach." He argues that for this reason, we must understand how teachers' mathematical beliefs influence instruction if we are to make progress in teacher education. In Ernest's 1989

work, he also makes the distinction between espoused (held) and enacted beliefs; a distinction that is still referenced today in research aimed at understanding teacher beliefs. The teacher's enacted beliefs on teaching and learning are always subject to the constraints of school and classroom contexts; there can be disparity between espoused and enacted beliefs due to these constraints similar to the Hoyles (1992) and Green (1971) place on social context (Ernest, 1989).

Ernest (1989) explains that teacher's mathematical beliefs generally fall into three categories:

- View or conception of the nature of mathematics,
- View of the of the nature of mathematics teaching,
- View of the process of learning mathematics.

Ernest's (1989) first category refers to a teacher's belief system regarding the nature of mathematics as a whole. Within this category, Ernest (1989) lists the instrumental view (mathematics as an accumulation of facts, rules or skills), the Platonist view (mathematics as static but unified body of specific knowledge—i.e., Algebra as consisting of a list of topics) and the third view can be described as a process of inquiry, ongoing formulation of ideas (with the possibility of complete revision). The view of the nature of mathematics teaching takes on three general forms: teacher as instructor, teacher as explainer, or teacher as facilitator, each of which to be discussed in detail shortly.

Ernest's 1989 work illustrates the critical components of the nature of beliefs and their relationship to practice. Ernest believed that the nature of mathematics is the critical factor in shaping teachers' beliefs of mathematics teaching and learning.

Several authors have expanded and revised Ernest's 1989 work, establishing the usefulness of his framework regarding mathematics teacher beliefs and actual teaching practice. Beswick (2005) provides such an overview in her table below, grounded in theory

(Lerman, 1983; Skemp, 1978). These relationships have since been demonstrated empirically in studies in the area of mathematics teachers' beliefs (Anderson & Piazza, 1996; Perry, Howard & Tracy, 1999; Thompson, 1984; Van Zoest et al., 1994). According to Beswick (2005), beliefs along the same row are considered theoretically consistent. Beliefs in the same column are regarded as a continuum on the following page.

TABLE 1. Beswick's (2005) Relationships between Beliefs.

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about learning (Ernest, 1989)
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content-focused with an emphasis on understanding	Active construction of understanding
Problem-solving	Learner-focused	Autonomous exploration of own interests

The first row of beliefs is generally thought to correspond with *traditional mathematics teaching* practices (Ernest, 1989; Beswick, 2005; NCTM, 2010). The traditional mathematics classroom, still prevalent today (Battista, 1994; Brandy, 1999; Hiebert, 2003), includes the memorization of facts as well as the ability to follow rules, execute procedures, and plug in formulas, with classrooms often teacher-directed. The teachers' role in traditional classrooms is to "provide clear, step-by-step demonstrations of each procedure, restate steps in response to student questions, provide adequate opportunities for students to practice the procedures, and offer specific corrective support when necessary," and the ultimate mathematical authority is the textbook from whence "the answers to all mathematical problems are known and found" (Smith 1996, p. 190). Traditional mathematics classrooms

are often cited as a cause of negative attitudes towards mathematics (Fennema & Sherman, 1994; McLeod, 1998).

The second row (and in some cases, the third row) of beliefs is most often associated with the phrase *reform mathematics* (Cobb, Wood & Yackel, 1990; Steffe, Cobb & von Glasersfeld, 1988). These beliefs are based on the assumption that children construct their own knowledge. Also, these beliefs are based in theoretical approaches to learning which have combined a constructivist orientation with a strong emphasis on social interaction. Teachers generally assume each student's production is personally meaningful and take responsibility for helping students verbalize these ideas in a mathematically meaningful way (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). The communication and behavior modeled by the teacher then becomes a model for student-student and student-teacher interactions. Students benefit and learn from sharing results, thus obtaining a better understanding of ideas and solutions, learn to refine solutions and learn other solution paths. Students also learn to defend, reason and argue their positions. Persistence is encouraged with an emphasis on process versus product. "Once one accepts that the learner must herself [*sic*] actively explore mathematical concepts in order to build the necessary structures of understanding, it then follows that teaching mathematics must be reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process" (Schifter & Fosnot, 1993, p. 9). This being said, there is still an agreed upon need for practice of mathematical skills (NCTM, 2000). The third row of beliefs involves teacher as facilitator of a shared community. Students independently explore their own interests and teachers intervene if requested by the student or if the teacher deems it necessary (Ernest, 1989).

An Illustration

The interaction between the nature of mathematics, teachers' beliefs on teaching and learning and the use of mathematics knowledge is best illustrated by the following two studies. First, Peterson, Fennema, Carpenter and Loef (1989) investigated the relationships among first-grade teachers' pedagogical content beliefs, teachers' pedagogical content knowledge, and student achievement in mathematics. In particular, the authors studied 39 first-grade teachers' beliefs about instruction, children's learning of mathematics and teachers' pedagogical content knowledge related to addition and subtraction. The authors used a cognitive science framework to define teachers' pedagogical content beliefs; specifically, what teachers believe about student learning in the following four areas: beliefs about how students acquire knowledge, how mathematics instruction should be organized, what the basis is for sequencing topics for instruction, and mathematical skills that should be taught in relation to understanding and problem-solving.

Peterson, Fennema, Carpenter and Loef (1989) found that teachers with a more cognitively based perspective made extensive use of word problems in introducing and teaching addition and subtraction in first grade with their students scoring higher on word problem-solving achievement than students of teachers with a less cognitively based perspective. Teachers with a more cognitively based perspective also used specific counting techniques (counting up, etc.) more extensively than teachers with a less cognitively based perspective. Using the same approach, Staub and Stern (2002), in a longitudinal study of 496 German elementary students, found that teachers with a constructivist orientation had students with larger achievement gains in mathematical word problems over two years. Specifically, teachers used structure-oriented tasks which required specialized teacher content knowledge in the area of number sense to create learning opportunities that foster

conceptual gains (Staub & Stern, 2002). These studies are two examples of few that have investigated teacher beliefs along with the use of mathematics content knowledge.

What is There Still to Learn about the Manifestation of Beliefs within Specialized Content Knowledge Use?

There is a tremendous amount to learn in this area of research, starting with simply understanding *what* beliefs occur when teachers use specialized content knowledge and *how* these beliefs are manifested in such knowledge in practice. The work above, although relevant in illustrating the general idea of teacher pedagogical content beliefs and content knowledge use, do not focus solely on specialized content knowledge in mathematics. Work done thus far is also limited in two additional ways:

- As discussed in Part I of Chapter 2, mathematics knowledge in teaching is typically studied in isolation without consideration of teacher beliefs. Similarly, teachers' beliefs are typically investigated without the consideration of mathematics knowledge use (Beswick, 2005). These two approaches limit the understanding of the beliefs-to-practice connection within a specific content domain.
- Studies involving teacher beliefs and mathematics teaching practice often measure beliefs in a single context risking not only a loss of richness in understanding of the role of beliefs in actual teaching practice, but also resulting in inconsistent results in how teachers use their beliefs (Beswick, 2005).

Beswick (2005) and others (Ernest, 1991; Raymond, 1997; Staub & Stern, 2002) argue for the investigation of teachers' beliefs within the domain of mathematics. Others working in the area of mathematics teachers' beliefs agree (Clark & Peterson, 1986; McLeod, 1998; Munby, 1982) explaining that although there is agreement that teachers' pedagogical beliefs make up an important part of teachers' general knowledge through which teachers' perceive, process and act upon information in the classroom. Research in this area has rarely referenced the knowledge of subject-matter upon which teachers' decisions are made. My

dissertation addresses this limitation by investigating the role of teachers' beliefs *specifically within the domain of mathematics*.

In regard to the second limitation, Beswick (2005) notes that when examining beliefs, since we must infer such beliefs from the words or actions of individuals and hence, according to Beswick, we can attach more certainty to a belief when the belief is evident in both the words and actions of the teacher within the realistic context of the classroom. Pajares (1992) agrees, explaining that since beliefs must necessarily be inferred from the words or actions of individuals, more certainty can be attached to the existence of a belief if evident in both. Therefore, it is suggested that multiple data sources be used to understand teachers' beliefs and their manifestation within practice (Beswick, 2005; Green, 1971; Handal, 1995; Pajares, 1992), particularly advising the use of classroom observation to verify the consistency of teachers' beliefs. My dissertation addresses this limitation by investigating the role of teachers' beliefs in multiple contexts including:

1. A beliefs survey, used as an initial indicator of beliefs,
2. An interview explicitly inquiring about beliefs and other factors typically associated with teachers' beliefs,
3. A structured think-aloud interview involving both specialized content knowledge and opportunities to explore the role of teacher beliefs,
4. Classroom observations to aid in uncovering consistency or inconsistency in teachers' beliefs in multiple contexts. Specifically, careful consideration is given to the specific context in which specialized content knowledge use occurred, to provide a rich description of the manifestation of teacher pedagogical content beliefs in specialized content knowledge use.

Lastly, as evident in the above discussions, there are a variety of terms used quite loosely and sometimes, in unclear ways. For example, the term pedagogical content beliefs and pedagogical content knowledge beliefs are confusing and require further clarification. Similarly, whether one is categorizing a belief as traditional or platonist or nontraditional or socio-constructivist is unclear. Hence, in Chapter 3, I will provide set definitions for the terms as used within this study.

Conclusion

The National Council of Teachers of Mathematics, as well as other entities promoting mathematics reform, recognize that the fate of the educational change and ultimately, mathematics teacher quality, lies within the reality that both teacher content knowledge and teacher pedagogical content beliefs are key factors in effecting change in the ways mathematics is taught and learned in schools (Battista, 1994; NCTM, 2000). I hope to continue to build upon the extensive work of the LMT research group in uncovering the specialized content knowledge used in quality mathematics teaching, as well as extend the this work to include the role of teachers' belief systems in how this knowledge is used. My practical goal is to provide the reader with a more cohesive picture of how specialized content knowledge is used in practice through multiple lenses.

CHAPTER 3

METHODOLOGY

In the first part of my dissertation, I focus on understanding the content and organization of middle school specialized content knowledge. This knowledge is vital to quality mathematics instruction, with quality instruction, defined in Chapter 2, as the holistic and balanced MQI dimensions that, taken together, comprise quality mathematics instruction (Hill, et. al, 2010). While the study of this topic is not new, reviewing the literature revealed several limitations. Most relevant is that conceptualizing and measuring mathematics knowledge for teaching (MKT) at the middle school level is relatively new work (Hill, 2007). Although there has been some success in identifying and differentiating between specialized content knowledge and common content knowledge at the middle school level, the differentiation the two is not yet clear (Hill, 2007). Hill (2007) tested a two-factor model on an MKT survey specifically designed for the middle school level. The results suggested a one-factor model with SCK and CCK as one overall construct. Despite these initial findings of a one-factor model, there is compelling research to suggest otherwise (Hill, Dean & Goffney, 2006; Hill & Ball, 2004; Hill et al., 2006; Hill, Rowan & Ball, 2005).

SCK is, in fact, a distinct type of knowledge, essential and important, specific to the profession of mathematics teaching. Although CCK is essential as well, SCK and CCK are two noticeably different types of knowledge that teachers hold, both contributing to quality mathematics instruction in basically different ways. A teacher's ability to calculate 24% of 38 is quite different from understanding if a student's method will work under all possible circumstances, noting that a student's method is often quite different from the teacher's.

Both skills are vital to quality mathematics teaching; the former requires CCK, the latter requires SCK.

I now begin to discuss the methodological choices underlying the approach to exploring my first research question: *What specialized content knowledge do middle school mathematics teachers hold? In particular, how does this knowledge differ from common content knowledge?* In doing so, I describe a series of steps of confirmatory factor analyses and the methodological purpose behind each step. I first discuss many items that must be addressed before a factor analysis study can commence.

The MKT Survey and Dataset

In this section, I examine the middle school MKT survey designed and administered by Hill in 2005–2006 (2007) and dataset resulting from administering the middle school MKT survey to a large, nationally representative group of middle school math teachers. The MKT survey tasks intended to measure common content knowledge and specialized content knowledge at the middle school level. A portion of the MKT survey was designed and developed during an earlier time period, for both the Study of Instructional Improvement (SII) and Learning Mathematics for Teaching (LMT) projects at the University of Michigan, starting in 1990, and was initially intended to measure mathematics teacher knowledge held by K–6 teachers in the following domains: Numbers and Operations, Ratio, Proportion and Percent, and Patterns, Functions and Algebra. Based upon some of this work, Hill (2007), along with a team of researchers, created a similar survey (using some of the questions designed earlier in the elementary MKT survey) aimed at measuring middle school teacher knowledge in the same domains. The resulting middle school MKT survey proposes to capture both common content knowledge (CCK) (i.e., the mathematical knowledge such as the ability to recall mathematical ideas and procedures correctly) as well as specialized

content knowledge (SCK) (i.e. teachers' mathematical ability to analyze student solutions and select grade-level and concept/activity-appropriate representations students can comprehend) *in the practice of teaching* (Ball, Schilling & Hill, 2004).

Data

Given my first research question, I sought data that could be used to describe specialized content knowledge held by middle school math teachers, including topics central to the Algebra curriculum (Common Core State Standards, 2009; NCTM, 2010). Hill (2007) had a dataset that fit this general description from her 2007 study that resulted from administering the middle school MKT survey to a large, nationally representative sample of United States middle school teachers in the years 2005-2006 (Hill, 2007). With weights applied, the sample population was demographically similar to the U.S. population of middle school teachers. Hill (2007) notes also that this sample is characteristically similar to another recent nationally representative sample of U. S., middle school teachers taken by the National Survey of Science and Mathematics Education (NSSME) in 2002.

Participants

In gathering study participants, Hill (2007) notes that schools were selected first from the National Center for Education Statistics (NCES) Common Core Database (CCD) for the school year 2002-2003. This publicly available database is the result of annual collection of data about all public schools and districts in the United States. The CCD data are supplied by state education agency officials and include information that describes schools and school districts, such as school name, address and phone number, as well as descriptive information about students, staff and other demographics (NCES, 2011). According to Hill (2007), the 1202 middle schools in the CCD Preliminary School Universe

file were then stratified first by region and urbanicity, then by probability proportional to the middle school's size. Since the structure of middle schools vary across the United States, Hill chose to define middle schools as those that had at least 10 students in grades 6-8. That is, data from schools with grades 5 or 9 were not used (Hill, 2007).

For each middle school chosen, teacher lists were obtained from Quality Educational Data (QED), a database which contains teacher lists of U. S. public schools nationwide. This accounted for 75% of the teachers resulting from the chosen schools in the participant pool. With some teacher lists inaccessible through the QED database, the selection of the other 25% was attempted by the University of Michigan Institute for Social Research (ISR) by contacting the remaining schools directly. The end result was 1065 schools, of which 1000 were chosen. For each of these schools, a teacher was chosen at random from middle school math teachers in grades 6-8 (Hill, 2007).

Distributing the Middle School MKT Survey and Gathering Results

Middle school MKT surveys were then sent to a *subset* of the larger teacher sample by mail, with teachers responding to the survey again by mail at a 64% response rate. It is important to note that teachers received a fifty dollar incentive to complete the survey (Hill, 2007). Hill (2007) reported that teachers received up to three mailings of the survey along with a reminder if needed. Hill (2007) suggests that the high response rate was due to these factors as well as teachers' affective responses to the survey experiences. That is, teachers reported both enjoyment of the survey and the thought provoking nature of the survey tasks (Hill, 2007). Hill (2007) then reported on this data, collected from April to June of 2005, in her 2007 article *Mathematics Knowledge of Middle School Teachers: Implications for the No Child Left Behind Policy Initiative*, where she describes the dataset and its associated study in more detail.

During the years 2005-2006, *additional* data was collected from the rest of the initial sample. In fact, a significant amount of data was gathered after her initial analysis. Hill (Personal Communication, March, 2011) suggested that an additional factor analysis should be done on the *full* middle school MKT Survey responses to verify the content and organization of common and specialized content knowledge reflected in the survey tasks. The MKT dataset was then obtained from Hill in 2011 by the author via e-mail. Hence, this study will perform similar confirmatory factor analysis techniques as Hill (2007) and other necessary analytic tools to understand the content and organization of specialized content knowledge as represented by MKT Survey tasks. It is important to note that with any secondary data set, especially with the passage of time from the year 2007, less information is available and will limit not only the similarity of techniques used in analysis but overall comparisons of results.

This dataset was appropriate for my first research questions primarily due to the middle school teacher population sample and middle school MKT Survey content. Additionally, the MKT survey has been used extensively in past research to successfully uncover, understand and categorize specialized content knowledge at the elementary and middle school level (Ball, 1990; Rowan, et.al, 2007; Hill, et. al, 2005; Hill, 2007).

Missing Data

According to Hill (2007), the final raw dataset contained four teachers with invalid codes for credential type, so these four data points were considered as missing. Again using the NCES data, Hill (2007) notes that she found no statistically significant differences in school size, number of classroom teachers, school Title 1 eligibility and place (rural vs. urban) when comparing participants with missing data to those without missing data. United States regions showed a marginally significant difference in response rates with the

Midwestern teachers more likely to respond (68%), and Western teachers less likely to respond (57%). Pupil-teacher ratios were marginally significant, with responders having slightly fewer students on average than nonresponders. These analyses were repeated with no significant differences in descriptive findings. Observations were missing from each variable, however, and in some cases, a variable's missing data number was significant. Deleting variables was not preferred due to the specialized nature of each task and the limited number of tasks.

However, is difficult to draw conclusions as to why certain data is missing, one of several limitations in using a secondary dataset. Removing observations can, at times, lead to biased results. I initially performed multiple imputation using MIANALYZE in SAS (2011), which basically creates several imputed values for each missing value. Each of the imputed values forms a distinct dataset upon which analysis can be done (King, et. Al., 2001; Little & Rubin, 2002; Rubin, 1987). In comparing the mean score on individual survey tasks with different results from imputed values, however, there were significant mean differences between the initial and newly proposed dataset for key survey tasks. Hence, for variables with missing observations, the mean value for respective MKT task were used in place of missing data.

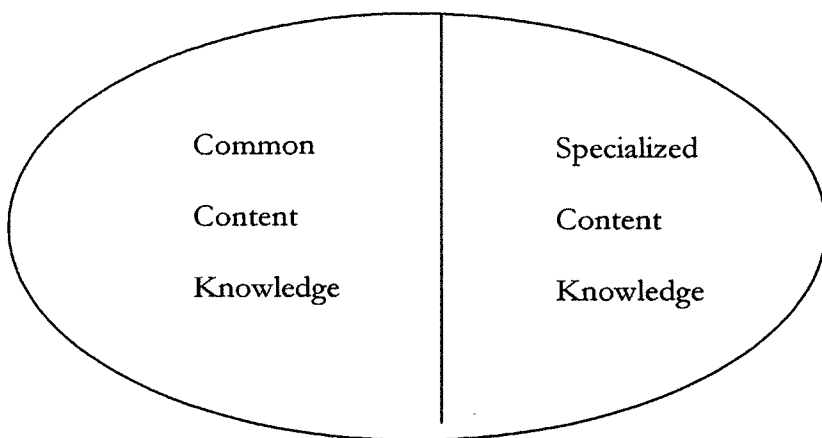


FIGURE 1. A subset of the proposed MKT Model (LMT Research Group, 2013).

A subset of the full MKT model is shown above and includes the constructs common content knowledge (CCK) and specialized content knowledge for teaching (SCK). Although SCK is the focus of this work, SCK and CCK are closely related. CCK can be thought of as mathematics knowledge typical of non-mathematics educators including, but not limited to, procedural knowledge, basic computations and comparisons with numbers. This knowledge and set of skills is not necessarily distinct from those held by other educated adults or even mathematicians. SCK is described as “the mathematical knowledge and skill unique to teaching” (Ball, Phelps, & Thames, 2008, p. 400). Teachers use a type of decompressed mathematical knowledge to look for patterns in student errors, examine student solutions to see if the solution will work in other similar situations, and provide an explanation as to why a particular student solution works or not (Hill, Rowan, & Ball, 2005).

An illustration of the difference between the categories CCK and SCK is provided in the 2008 article authored by Ball, Thames’ and Phelps. A teacher, Ms. Daniels, had a mathematics background including two years of Calculus, a course in mathematical proof writing, four computer science courses and modern algebra course. She was able to correctly divide fractions. Although Ms. Daniels was able to divide fractions, she was unable to

provide a correct representation for dividing fractions to her students or explain why the “invert and multiply” algorithm works. Although necessary knowledge for mathematics teaching and learning, CCK is far from sufficient for quality mathematics teaching.

A History of the Development of the Measures of CCK and SCK

The original elementary school level MKT survey items, some of which were used by Hill (2007) when developing her middle school MKT survey, were written by a team of professionals including mathematicians, teachers and math educators, and psychometricians averaging 20 or more years of experience, to reflect mathematical knowledge used in the actual practice of teaching (Ball, Schilling & Hill, 2004). There were initially 138 items developed which were further subdivided into four topic types (number concepts, operations, and patterns, functions, and algebra).

Expanding the MKT Work to Assess Middle School Teachers' Mathematical Knowledge

Most of the item-writing had been done at the elementary level and hence it was necessary to select items appropriate for the middle school level (Hill, 2007). Of the items previously written, two topics were chosen for the construction of the middle school survey: (1) numbers and operations and (2) patterns, functions and algebra. Math educators, mathematicians, professional developers, the research project group and current and former teachers constructed additional middle school survey items (Hill, 2007). The majority of the item-writers were the same professionals who wrote the initial elementary level MKT survey items. Keeping in line with the elementary school work, item writers drew on their extensive experience in and knowledge of teaching mathematics, cutting edge research on mathematics knowledge used in teaching, and observing classroom instruction while writing and reviewing items over a year's period of time (Hill, 2007).

The two topics of numbers and operations, and patterns, functions and algebra were chosen for several reasons. An estimate from the Third International FIX (according to Hill, 2007) shows that 40% of eighth grade lessons in the U.S. focus on numbers and operations (Peak, 1996). Concepts included in Patterns, Functions and Algebra also play a critical role in the middle school years. Hence, the 2005 MKT Survey resulted in 92 items and fell approximately evenly between the two concepts, numbers and operations and patterns, functions and algebra, in specific areas such as: whole, rational and integer numbers and operations, ratio, proportion and percent, radicals, linear, quadratic and exponential functions, Algebraic expressions and equations, absolute value and inequalities (Hill, 2007).

Items were ultimately scored and responses classified as “correct” (the value in this case is 1) or “incorrect” if any of the other incorrect responses were chosen. That is, variables were ultimately dichotomous. This was the technique used by Hill (2007) and hence for consistency, this approach was also used here.

Reliability and Validity

Pragmatically speaking, one of the most significant arguments for the reliability and validity of the middle school MKT survey tool has been the extensive work done over the time period of 2003 until currently involving the elementary school MKT survey tool, demonstrating reliability and validity in multiple nationally representative samples and in multiple research contexts. That is, there has been extensive work done by the LMT and SII research groups modeling the mathematical knowledge elementary school teachers hold, as well as extensive work in measuring this knowledge using the MKT survey, including linking teachers’ MKT to student learning (Rowan et al., 1997; Hill, et al., 2005; Ball, et al., 2005; Hill 2007). Reliability and validity of the MKT survey is well established at the elementary school level in multiple studies of both a quantitative and qualitative nature (Hill, Ball, Blunk,

Goffney & Rowan, 2006; Hill, Dean & Goffney, 2006; Hill, Dean, et al., 2006). A weakness of this work is the inability to disclose specific survey tasks. However, disclosing MKT tasks is clearly inappropriate considering the use and further development of the tool.

How the MKT middle school tasks were chosen speaks to the reliability and validity of the survey. In regard to the reliability of the middle school MKT survey items, more than one-fourth of items appropriate for the middle school level directly from this elementary MKT survey *and* specifically chose items representative of core topics at the middle school (i.e., Numbers and Operations and Patterns, Functions and Algebra). In terms of validity, items on the middle school MKT taken directly from the original cognitive interviews were used to previously establish content validity. Additionally, two items were taken from studies that link teacher knowledge to student achievement (Hill et al., 2005; Rowan et al., 1997). Any additional items added to the MKT survey item was written by largely the same item writers. Items were also of the same format and style.

Multiple research techniques were used to establish content validity at the elementary school level tasks relevant to the middle school items are discussed here. First, in establishing content validity, cognitive interviews were used to determine that teachers' answers do reflect their underlying mathematical thinking (Hill, Ball, Blunk, Goffney & Rowan, 2006; Hill, Dean & Goffney, 2006). Additionally, the quality of teachers' instruction has been shown to correlate with their MKT measures using actual video of classroom instruction (Hill, Dean, et al., 2006). Much of the work done by the LMT research group over the years involved the use of psychometricians and teams of statisticians, as well as mathematics educators at all levels of instruction. In some cases, as with the cognitive interviews, other groups of individuals, such as professionals with mathematics knowledge,

but with no teaching experience, were used to validate the existence of SCK as a special type of knowledge unique to mathematics teachers (Hill, 2007).

Cognitive interviews offered additional information on reliability and validity of MKT survey items. Cognitive interviews uncovered that a teacher's reasoning for a particular item is consistent with the multiple choice item he or she selects (Hill, Dean & Goffney, 2008). Higher MKT survey scores correlated with higher-quality mathematics instruction during teacher observations of over 30 teachers. For mathematicians in particular, mathematicians performed better on CCK items than SCK items, providing proof of the claim that SCK knowledge is specifically tied to the activities of K–12 teaching (Hill, Dean & Goffney, 2008).

It is likely that much of this validity information resulting from the elementary school generalizes to the middle school teacher population for the above stated reasons. Additionally, the same construct model was used in reliability and validity of both the elementary and middle school MKT survey items. Items chosen factored into either CCK or SCK categories in previous work at the elementary school level (Rowan, et al., 1997; Hill, et al., 2005; Ball, et al., 2005; Hill 2007), indicating a strong possibility of a similar situation at the middle school level.

Setting the Stage for Analysis

In previous work done by Ball, Phelps, and Thames (2008), one of the most significant findings was the result of extensive factor analysis indicating that content knowledge and specialized content knowledge at the elementary school level were related, but yet not completely equivalent. When Hill (2007) analyzed the middle school MKT survey results, this delineation was not as clear; scales created to represent SCK and CCK theoretical constructs “were correlated at .79, where .81 would be a perfect correlation,

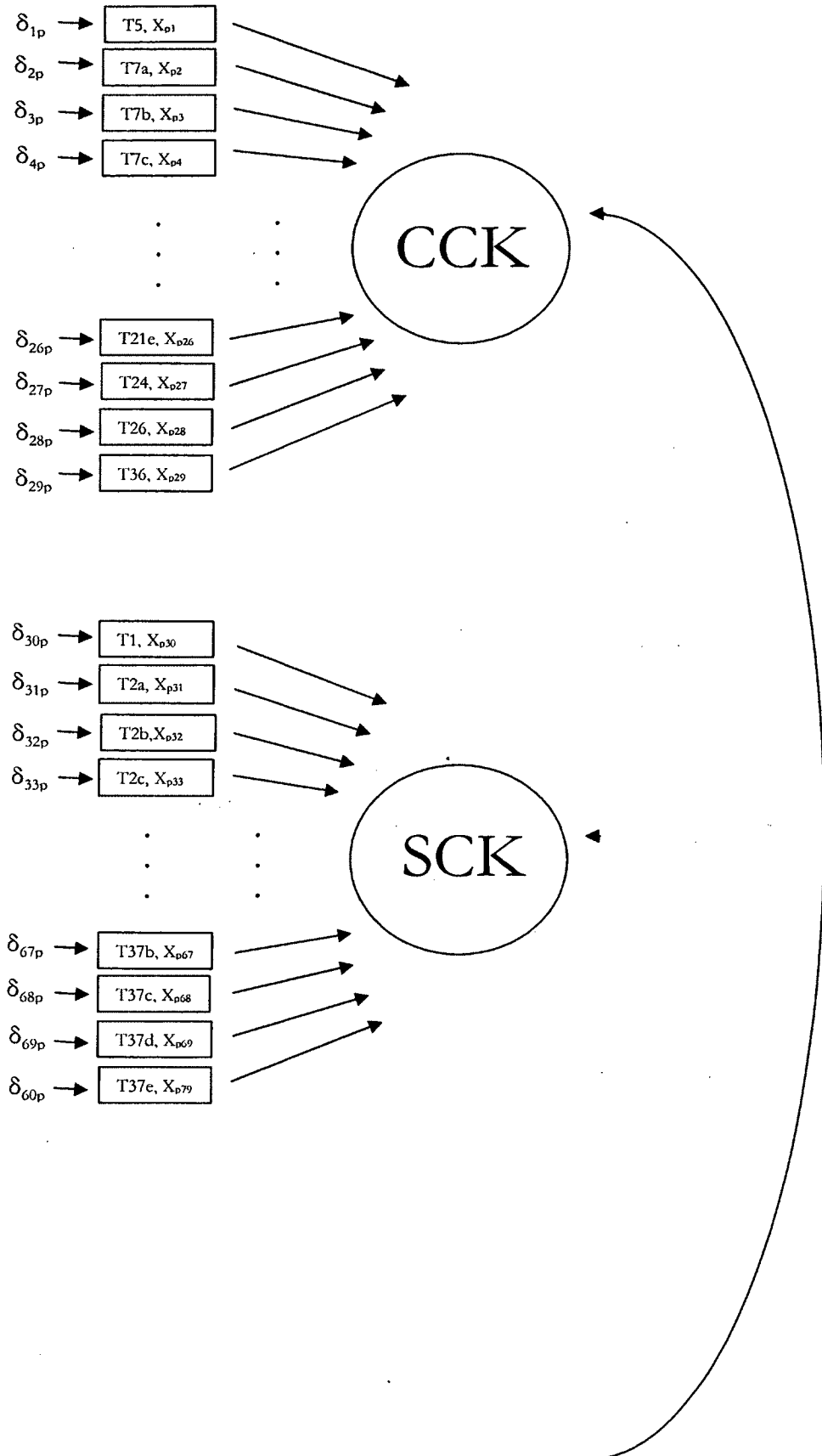
accounting for measurement error. These strong correlations, along with the factor analysis results, suggest a one-factor model” (p. 73).

Factor analytic procedures can be useful in attempting to differentiate between two distinct constructs. Such procedures are useful and used extensively in educational measurement as a way of understanding relationships between observed variables and a typically small number of potential latent factors. This work can be expanded to include the possibility of correlations amongst the factors being explained by a more global factor or factors, referred to as global or second-order factors. In this study, I first repeat the confirmatory factor analysis done by Hill (2007) on the model she proposed, a two-factor model including the factors CCK and SCK, Model 1 shown below. Also note the individual number of each of the CCK or SCK tasks.

Table 2. The Organization of MKT Survey Tasks for Model 1.

SCK	CCK
1	5
2	7a-7d
3a-3d	8a-8f
4	9
5	11a-11c
6	12
10	13
16	
17	14
18a-18d	15
20	19a-19d
22a-22d	21a-21e
23	24
25	26
27	36
28	
29	
30a-30c	
31	
32	
33	
34	
35	
37a-37e	

Figure 2. The Proposed Model 1.



Although I cannot reveal the content of the tasks abiding by the various agreements in using the MKT survey tool, it is important to give examples of tasks to provide the reader with a deeper understanding of task content and organization. Below are two example tasks: one representative of CCK, the other representative of SCK. The specific multiple choice selections were not included.

CCK. Mr. Nadeau is looking at a textbook problem he plans to give his students. Help Mr. Nadeau decide which of the following four choices is a correct solution to the following equation involving one variable.

SCK. Mrs. Staples is the following proportion equivalencies. Her students have not yet learned the procedure or concepts behind cross-multiplication. Considering her students' background in this sense, which of the four proportion equivalencies below would be hardest to solve for her students?

I noticed specific tasks initially classified as SCK, which upon further inspection, appeared to be accessing teachers' CCK. For example, consider a survey item that begins with a description of a setting where students discuss an equation of one variable. In this discussion, students offer different statements that constitute the possible correct answers to solving the equation of one variable. At first glance, the task may seem to be measuring SCK, if the task required analyzing student solutions using specific specialized content knowledge. The task could require the teacher simply solving the problem, accessing primarily CCK.

As I continued to review items, I also noticed a possible different organization of the MKT survey tasks. Specifically, CCK items fell into two seemingly distinct categories: procedural knowledge (knowing how, mathematically) and conceptual knowledge (knowing why, mathematically). (It is important to note that procedural and conceptual knowledge are

accessed simultaneously as well). SCK items also fell into two categories: choosing mathematical scenarios (CMS) and evaluating student claims (ESC). Although these categories seem to be describing a task, in reality, they describe SCK; SCK *embedded* within the tasks of choosing mathematical scenarios and evaluating student claims. This very important distinction will be discussed in more detail in Chapter 4. Examples of each are given below.

PK Task 21. Mr. Nadeau is looking at a textbook problem he plans to give his students. Help Mr. Nadeau decide which of the following four solutions is a correct solution to the following equation involving one variable.

CK Task 3. Ms. Ellen asked her students to explain why the number 1 is not prime. Help Ms. Ellen inspect the four choices and decide the best explanation.

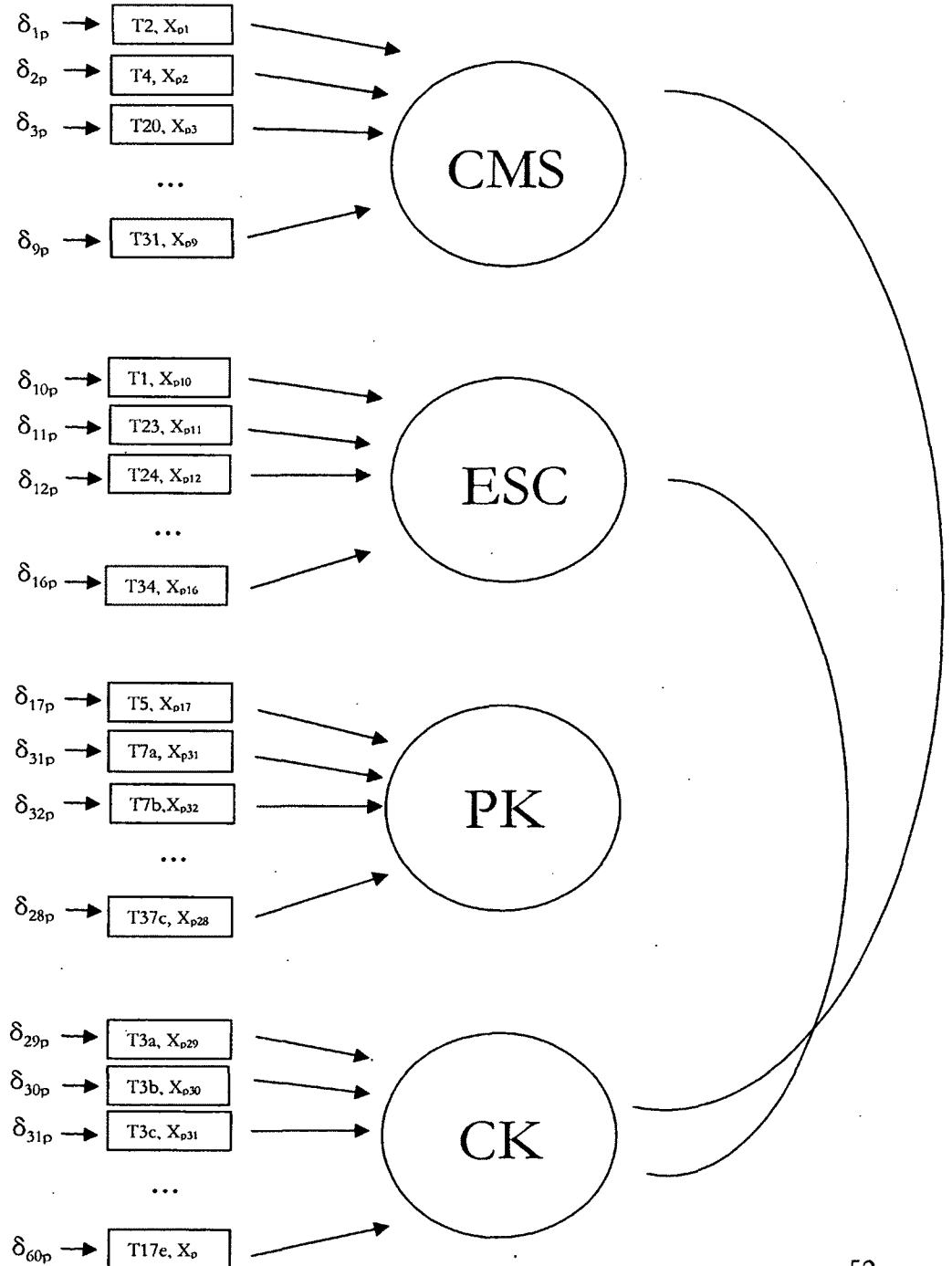
CMS Task 31. Mrs. Staples is the following proportion equivalencies. Her students have not yet learned the procedure or concepts behind cross-multiplication. Considering her students' background in this sense, which of the four proportion equivalencies below would be hardest to solve?

ESC Task 34. Mrs. Doubtfire is reviewing class work while walking around her classroom and working with students. A group of four students has four different methods for finding 30% of any positive real number, x . Help Mrs. Doubtfire look at the four student responses and determine if she can tell the student if their method will work for any positive real number.

It became clear that the current MKT model might need some additional modifications. Hence, I considered what analytic techniques might be necessary. In addition to confirmatory factor analyses on Model 1, I considered a different model, incorporating the different organization I noticed. Model 2 resulted from several changes to

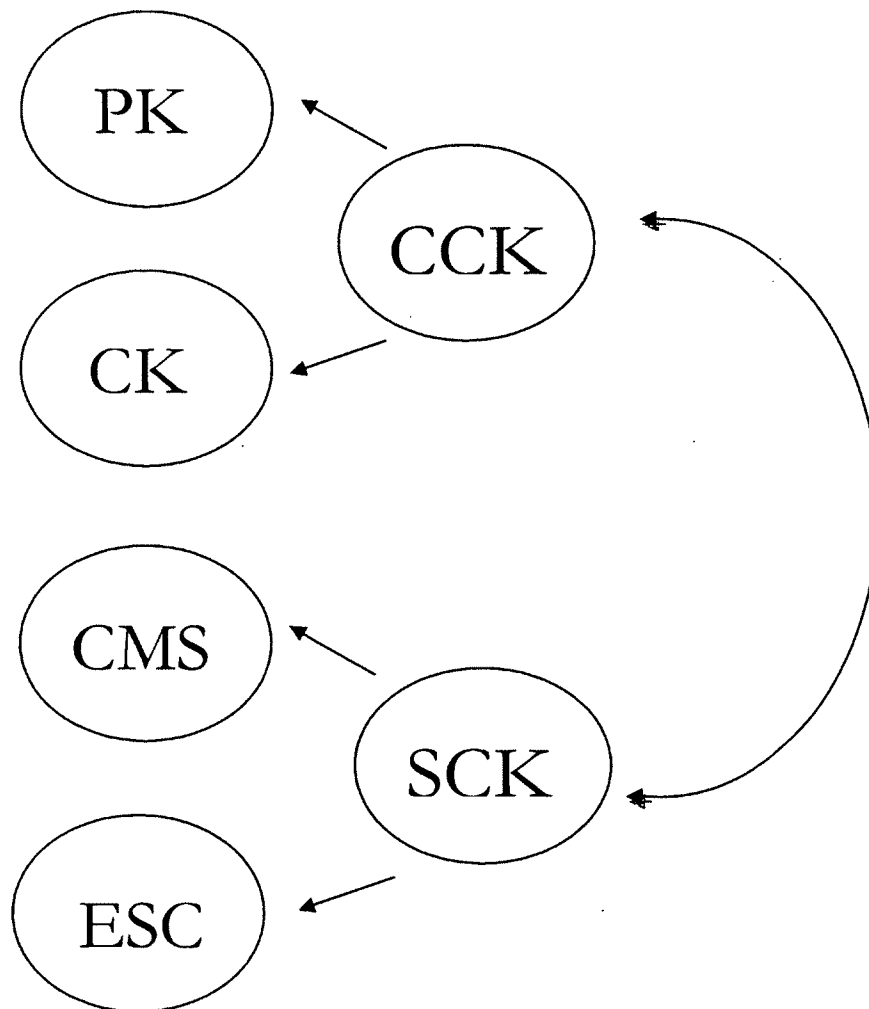
Model 1. Two changes were made. First, some items initially classified as SCK that were clearly CCK upon inspection were moved to the CCK category. Second, the tasks were then organized into PK, CK, CMS and ESC accordingly.

FIGURE3. The Proposed Model 2.



Lastly, a third second-order model is proposed with the addition of CCK and SCK as global factors. The importance of CCK and SCK is theoretically clear and hence the addition of this hierarchical structure is reasonable. Additional modifications were made based upon the results of Model 2.

FIGURE 4. The Proposed Model 3.



Lastly, a cross-validation of the initial and final models was conducted. The size of the initial sample allowed for such a technique to be used. When numerous modifications are made to an initial model, cross-validation is suggested (Cliff, 1983; Cudeck & Browne, 1983;

Benson & Bendalos, 2010). When performing cross-validation, correlations among the parameter estimates would be high across the two samples if the parameters of the respective models are generally found to be similar in magnitude. All models were tested using the statistical software package Mplus (CITE). Lastly, all three models (Model 1, Model 2, and Model 3) were compared and differences discussed.

Summary

Researchers are often interested in variables that cannot be observed directly. One can gain insight into latent factors or constructs through observed variables such as, in this case, survey responses hypothesized to measure the constructs CCK or SCK. CFA is a theory-driven technique. That is, the planning of CFA is driven “by the theoretical relationships among the observed and unobserved variables” (Schreiber, et al., 2006). Due to the limited work that has been done in testing the MKT model on middle school data, this immediately calls into question the reliability of the results, including the stability of parameter estimates. However, it is important to continue to test the theoretical model, particularly in the case of Model 1, to provide further insight into Hill’s 2007 results. In summary, the analysis will proceed in multiple steps, testing a series of three distinct confirmatory factor models, each of which will be shown in more detail in Chapter 5, Analysis and Findings for Part I of the dissertation. I now turn to the expansion of this work, the Analysis and Findings of Part II of the study, investigating MKT use within a broader context of the classroom and the role of teachers’ pedagogical content beliefs.

II: Understanding How Teachers' Beliefs are Manifested in the Classroom Use of Specialized Content Knowledge

In the first part of this study, confirmatory factor analysis and related analyses were done to understand and differentiate between different categories of mathematics knowledge used in the practice of teaching, namely common content knowledge (CCK) and specialized content knowledge (SCK). SCK is specific to the profession of mathematics teaching; the subtleties of teaching and learning of mathematics. For example, teachers not only need to know the properties of rectangles and communicate this to students (CCK); teachers must also know what representations of a rectangle would be most helpful in teaching these properties, decode properties in such a way that students can understand, and find a mathematically accurate definition of a rectangle that is accessible to all learners. This clearly important type of teacher knowledge is the focus of Part II of this study, yet with an important additional consideration, the role of teachers' pedagogical content beliefs in the use of such knowledge. Teachers' pedagogical content beliefs are an *additional type of teacher knowledge* found to significantly influence teacher practices (Ernest, 1989; Hill, 2007; Hill, Ball & Schilling, 2008). Specifically relevant to this study, teachers' pedagogical content beliefs are thought to play a substantial role in shaping how mathematics knowledge is used in practice (Beswick, 2008; Buehl & Fives, 2009).

Few studies have investigated teachers' pedagogical content beliefs specifically with regard to specialized content knowledge due to the highly complex nature of both SCK and teachers' belief systems. This is troubling due to the fact that even teachers with a high level of mathematics content knowledge have *very* different beliefs about teaching practice and hence *significantly* different teaching behavior in the classroom (McLeod, 1998; Staub &

Stern, 2002). If we have only an understanding of teacher's SCK, we do not fully understand what is happening in the mathematics classroom. Hence, understanding the complex interplay among content knowledge, pedagogical content beliefs and what happens in the classroom is a focus of this study.

Due to the complexities surrounding the understanding of beliefs and SCK use, I chose a qualitative research approach considering its practical success in framing the exploration of highly complex questions. It is through the range and flexibility of multiple qualitative research techniques that I hoped to provide descriptions rich and explanatory in nature, truly capturing some of the complexities of teachers' beliefs and SCK use. I now turn to the specificities of the methodological basis for the second research question of my study: *How are middle school Algebra teachers' pedagogical content beliefs manifested in mathematics knowledge use in teaching?*

Introduction

Teachers' mathematics content knowledge use, beliefs and practices have been investigated both individually and collectively through a variety of methodological approaches. Conceptualization of each of these areas remains a challenge for researchers. For example, there is no agreed upon definition of beliefs and even less agreement as to how the area of teacher beliefs can be effectively conceptualized (Ernest, 1996; Handal, 2003; McLeod, 1998; Raymond, 1997). Due to the broad and complex conceptualizations in research over the years, as well as the complexity that arises from the interplay of teachers' beliefs and the use of SCK in the classroom, I chose a case study strategy to narrow the focus of *what* will actually be studied. The quintessential characteristic of case studies is that they strive towards a holistic understanding of systems of action, i.e. specialized content

knowledge use in action yet allow for a specific focus on certain aspects of the system, such as SCK and teacher beliefs.

A Pragmatic Approach to the Extension of a Previous Study

The decision to use the strategy of case study was made under the umbrella of pragmatism.

A pragmatic justification emphasizes the applied nature of case study research. As a method it can be advocated on grounds that it is more useful, more appropriate, more workable than other research designs for a given situation. Knowledge produced by case study would then be judged on the extent to which it is understandable and applicable...a pragmatic conception of truth undergirds this approach. (Merriam, 1988, p. 20)

Pragmatically speaking, since the Hill, Dean and Goffney (2005) study had similar goals to my own study, with respect to SCK, a similar methodological approach was used here with the additional consideration of teacher beliefs. Specifically, Hill, Dean and Goffney's (2005) MKT survey validation efforts included methodological choices that proved in the past to be fruitful in uncovering, understanding and describing specialized content knowledge. Although Hill, Dean and Goffney (2005) did not explicitly state the use of case study as a strategy, their work is similar to my own in several ways. In their work, a small sample of participants were chosen to take the MKT survey. These teachers were then interviewed in what the authors refer to as Sudman, Bradburn and Schwarz's (1996) concurrent think-aloud protocol. That is, teachers reported their thinking process while answering MKT survey tasks involving specialized content knowledge (Hill, Dean & Goffney, 2005). Hill, Dean and Goffney (2005) used this work to understand and describe the domain of mathematics knowledge used in teaching, with a distinction among topics in mathematics (i.e. numbers and operations, algebra) and the types of knowledge teachers hold (content knowledge, which is common to both mathematics and society and *specialized*

content knowledge, knowledge specific to the work of teaching) at the elementary school level.

As in the work of Hill, Dean and Goffney (2005), I also used MKT tasks with think-aloud techniques to understand how teachers use SCK, although the tasks I chose were middle-school level tasks. Additionally, due to the significant role of beliefs in shaping mathematics knowledge use in practice, I investigated the nature of teachers' beliefs and SCK in multiple contexts as suggested by prominent researchers in the field of teacher beliefs (Greene, 1981; Pajares, 1992; Beswick, 2005). Specifically, I designed a five-part study, including five distinct data sources, aimed at addressing the complexities surrounding the use of SCK, teacher pedagogical content beliefs and the often unpredictable beliefs-to-practice connection. These five parts are discussed in detail next, in Sections A–E below. It is important to note that it was a goal to have each participant complete all five parts of the study within one week, preferably within three days. At the conclusion of the study, all participants with the exception of one with significant extenuating circumstances, were able to complete the study within this timeline.

Choosing a Participant Pool

Teachers were chosen from the states of Massachusetts, and New Hampshire. Teachers were initially contacted by a New Hampshire Teachers of Mathematics (NHTM) e-mail list and asked to participate in the following five-part study described in detail below (see Appendix for consent forms and a full set of IRB documentation). Fourteen teachers responded initially. Of the fourteen teachers expressing interest in the study, ten teachers were chosen based upon their willingness to continue to participate in the remainder of the four parts of the study discussed next. A fifty-dollar compensation was provided at the

completion the study. This choice was made due to the difficulties of recruitment when a pilot study was attempted, but not completed, in the previous school semester.

The choice of participants was not deliberate. In other words, other characteristics such as gender, age and so forth were not a consideration. “They may be similar or dissimilar, redundancy and variety each having a voice. They are chosen because it is believed that understanding them will lead to better understanding...about a still larger collection of cases” (Stake, 1994, p. 237). Hence, each participant was unique and had something worthwhile to reveal through their ultimately individualized responses.

Embracing this variability is the strength of using a qualitative lens. The ways in which participants construct task and interview responses was in fact highly variable and specific to a particular participant, each participant contributing to an overall picture of different teachers’ beliefs and classroom practices encompassing the use of specialized content knowledge.

Section A – The MKT Survey

In Section A of the study, teachers participated in an approximately one-hour, on-line multiple choice self-report survey, the Mathematical Knowledge for Teaching (MKT) Instrument developed by the Learning Mathematics for Teaching Project (Hill, Schilling & Ball, 2004), tasks encompassing the mathematical knowledge used in the practice of teaching, including specialized content knowledge. The MKT survey typically takes approximately one hour to complete. Teachers could only take the survey once. However, if computer problems occurred, the teacher could go back and start the survey at the previous stopping point; however, this occurrence was not reported by participants. Teachers received a detailed document to aid in accessing the survey (located in Part II of the Appendix).

The MKT survey tasks specifically fell into three content areas (3 sub-sections of the survey): middle school numbers and operations (20 questions), and both elementary and middle school patterns, functions and algebra (17 questions each). Administering this survey served several purposes, one of which was to expose participants to the MKT practice-based mathematical tasks encompassing specialized content knowledge, similar to tasks later used in the structured interview where teachers talked through similar tasks in more depth in Section C of the study. Additionally, the survey results provided an idea of the level of MKT each participant held, which included the level of specialized content knowledge.

The choice of the use of the MKT survey versus other surveys designed to measure mathematics knowledge for teaching is multi-fold. The work of the LMT research group has shown that the MKT survey generally produces reliable scores in empirical studies. Furthermore, recent validity studies have shown that the MKT predicts instructional quality as measured by the Mathematical Quality of Instruction (MQI) instrument (Hill, et. al., 2008), and at least one study has reported a direct relationship between teachers' MKT scores and student learning gains (Hill, Rowan, & Ball, 2005). In theory, if a teacher scores in the high range of an MKT score, the teacher's quality of instruction should also be high and should theoretically reflect an integrated use of SCK.

Section B – Beliefs Survey

Teacher beliefs about teaching and learning were assessed by a questionnaire used by Staub and Stern (2002) building upon earlier work done in the area of pedagogical content beliefs by Fennema, et al. (1990), each of which has done extensive work in the area of pedagogical content beliefs in the domain of mathematics. An additional scale was added by Staub and Stern (2002) which was initially designed by Cobb et al. (1991). This resulting combination consists of a survey encompassing cognitive constructivist orientation of

teachers' pedagogical content beliefs, and other teacher belief orientations most commonly identified in mathematics education research, discussed in more detail below (see Table 1 from Beswick, 2005), explicitly about the teaching and learning of mathematics. Within this framework are four subscales: the role of the learner (beliefs about learning), relationship between skills, understanding, and problem solving (beliefs about teaching), socioculturalism (beliefs about teaching) and lastly, the role of the teacher (beliefs about teaching). The questionnaire consisted of four categories of items and teachers responded on a 5-point Likert scale: strongly agree, agree, undecided, disagree, or strongly disagree. Below, Table 5 describes each of the four subscales, which contains 12 items each (Staub & Stern, 2002).

Table 3. Beliefs Survey Subscales (Staub & Stern, 2002)

Subscale Title	Subscale Description	Subscale Example
Subscale I: The role of the learner (beliefs about learning)	Whether teachers hold the belief that children construct their own mathematical knowledge versus the belief that learning is primarily by transmission	Students learn best when figuring out ways to find answers themselves. Students learn best by following teacher examples.
Subscale II: Relationship between skills, understanding, and problem solving (beliefs about teaching)	Assessing the view that mathematical skills should be taught in relation to understanding and problem solving and the view that mathematical skills should be taught as discrete components	Students should have informal experiences with mathematics before memorizing facts. Students should be taught mathematical facts as a basis before exploration.
Subscale III: Socioconstructivism (beliefs about teaching)	Differentiate between teachers' pedagogical beliefs about teaching that are compatible with behaviorism and beliefs that are compatible with socioconstructivism	Given the right tools, students should be able to create their own methods for calculating. Quality teaching relies on the teacher demonstrating the right way to do a problem.
Subscale IV: The role of the teacher (beliefs about teaching)	View that mathematics instruction should be organized to facilitate children's construction of knowledge with the view that mathematics instruction should be organized for clear presentation	Students should be allowed to create solutions to word problems before the teacher shows them how. Students learn best when showed problem solving before attempting the problems themselves.

The survey reliability and validity was first addressed extensively by its original authors (Peterson, Fenema, Carpenter & Loef, 1989) by analyzing the face, construct, and criteria validity via a variety of experts in the field of mathematics education. In terms of reliability, the authors provide details of the internal consistency of both the subscales and overall scores using Cronbach's alpha on a sample of 39 teachers. The internal consistency was high, with the internal consistency of the total belief survey scale being .93. Their results on the three subscales showed beliefs about teaching and beliefs about learning to be highly

correlated. The focus of *this* study is on a subset of pedagogical content beliefs, specifically, beliefs about teaching mathematics. Due to the overlap and correlation between certain belief categories, however, the use of this particular survey was necessary. In terms of reliability, the survey itself has been used since 1989, over a period of twenty years, in similar studies, demonstrating similar findings as with the initial Peterson, Fenema, Carpenter & Loef (1989) survey (Fennema, et al., 1990; Cobb et al., 1991; Staub & Stern, 2002), with each of the authors addressing and improving upon the validity and reliability of the survey. For this study, the beliefs survey's main purpose is to give an initial indication of teacher beliefs about teaching mathematics, which will then be expanded upon in the focused interview, Section D of the 5-part study.

Although the survey encompassed indications of a constructivist orientation of teachers' pedagogical content beliefs, it is not the goal of the researcher to identify a specific teaching philosophy or to promote a constructivist versus traditional approach to mathematics teaching. Furthermore, items in the survey functioned beyond this capacity. When reviewing the specific items, there was clearly a map between survey items and the below three categories created by Beswick (2005), including instrumentalist, platonist and problem-solving belief orientations. Survey responses served as a indicator on a continuum, placing a participants beliefs within one of the three categories. The below categorizations served as guidelines for classifying teacher responses; which will later become more clear in the actual survey analysis. This not surprising, however, considering much of the work in mathematics education on teachers' pedagogical content beliefs generally focus on the categories of instrumentalist, platonist and problem-solving as primary categories in which mathematics teacher beliefs generally fall (although the names of the categories are not always consistent) (Ernest, 1991; Pajares, 1992; Beswick, 2005).

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989)
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of discrete knowledge, emphasis of performance
Platonist	Content-focused with an emphasis on understanding	Active construction of content understanding
Problem-solving	Learner-focused	Autonomous exploration own interests, social construction of knowledge

TABLE 4. An Expansion of Beswick's Relationships between Beliefs (Beswick, 2005).

Raymond (1992) completed a comprehensive study involving pedagogical content beliefs in mathematics, developing a criteria that was successfully used in gaining knowledge about what types of beliefs mathematics teachers hold. This list, based upon earlier work by the NCTM (2000) as well as Beswick's continuum and Ernest's categories of beliefs, is included here below. This list was used in the analysis phase to categorize beliefs indicated in the beliefs survey and focused interview.

Table 5.Criteria for the Categorization for Teachers' Beliefs about Mathematics Teaching

Traditional

- The teacher's role is to lecture and to dispense mathematical knowledge.
- The teacher's role is to assign individual seatwork.
- The teacher seeks "right answers" and is not concerned with explanations.
- The teacher approaches mathematical topics individually, a day at a time.
- The teacher emphasizes mastery and memorization of skills and facts.

- The teacher instructs solely from the textbook.
- Lessons are planned and implemented explicitly without deviation.
- The teacher assesses students solely through standard quizzes and exams.
- Lessons and activities follow the same pattern daily.

Primary traditional

- The teacher primarily dispenses knowledge.
- The teacher primarily values right answers over process.
- The teacher emphasizes memorization over understanding.
- The teacher primarily (but not exclusively) teaches from the textbook.
- The teacher includes a limited number of opportunities for problem-solving.

Mix of traditional and socio-constructivist

- The teacher includes a variety of mathematical tasks in lessons.
- The teacher equally values product and process.
- The teacher equally emphasizes memorization and understanding.
- The teacher spends equal time as a dispenser of knowledge and as a facilitator.
- Lesson plans are followed explicitly at times and flexibly at others.
- The teacher has students work in groups and individually in equal amounts.
- The teacher uses textbook and problem-solving activities equally.
- The teacher helps students both enjoy mathematics and see it as useful.

Primarily nontraditional

- The teacher primarily facilitates and guides, with little lecturing.
- The teacher values process somewhat over product.
- The teacher emphasizes understanding over memorization.
- The teacher makes problem solving an integral part of class.
- The teacher uses the textbook in a limited way.

Nontraditional

- The teacher's role is to guide learning and pose challenging questions.
- The teacher's role is to promote knowledge sharing.
- The teacher clearly values process over product.
- The teacher does not follow the textbook when teaching.
- The teacher provides only problem-solving, manipulative-driven activities.
- The teacher does not plan explicit, inflexible lessons.
- The teacher has students work in cooperative groups at all times.
- The teacher promotes students' autonomy.

In 1989, Petersen criticized research on teachers' beliefs as focused on general beliefs about teaching and learning, without taking into account the role of content knowledge and its use. Other researchers in this area have since echoed this concern (Pajares, 1992;

Beswick, 2005). This limitation was addressed with this study by both Section A and Section C, discussed next.

Section C – The Structured Interview

Each participant, after completing the MKT survey, participated in a structured think-aloud video-taped interview as soon as possible after completing the MKT survey. In this interview, participants talked through at least six MKT tasks representing core Algebraic knowledge in the domain of Patterns, Functions and Algebra (i.e. a structured task) similar but not identical to the MKT survey tasks. All six tasks involved specialize content knowledge. Two of the six tasks given in the first interview are shown here below, taken from the 2008 Released MKT Items (Ball, Schilling & Hill, 2004).

Task 1.

Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions $a - (b + c)$ and $a - b - c$ are equivalent. Provide as many explanations as possible as to how you would explain this to a middle school Algebra student.

Task 2.

Consider $-x < 9$. Marcie solved this problem by reversing the inequality sign when dividing by -1 so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number. How would you explain why this is so, if you consider a middle school Algebra student? Can you provide more than one explanation?

Although participants were teachers who are generally highly verbal (Charters, 2003), participants sometimes need some sort of communication or verbal cues during the interview process. For example, participants without instructions or practice may not report their thought processes frequently or thoroughly enough to meet the researcher's needs (Charters, 2003). Participants in a think-aloud study, however, are not to be coached or guided due to the potential bias that would be introduced in the resulting data (Ericsson & Simon, 1980). Participants should be allowed to focus on spontaneously (as much as this is possible) communicating their inner thought processes. Such verbal reports which follow very rapidly after a thought process are generally accurate and reflect conscious thought, allowing the researchers to focus on the participants' immediate awareness, not delayed explanations for their actions (Cooper, 1999, Olson, Duffy & Mack, 1984). In order to design think-aloud techniques to reflect address the above issues and allow for natural thought processes, the researcher must avoid influencing the participant, hence prompting during the structured task interview and the above other issues were adopted and used during this phase of data collection.

For these reasons, less intrusive techniques were used that tended to influence participants minimally and only if necessary. One such technique included a 'please keep talking' hand signal used as to not interfere with the participants thought process, as suggested by Sugirin (1999). Participant pauses and omissions may reveal insights that are just as informative as verbal responses and hence the 'please keep talking' hand signal was used primarily when participants wanted to go onward in verbalization but are unsure if it was allowable, not during silent periods, short pauses or omissions as suggested by Pressley and Afflerback (1995) among other researchers in the area of think-aloud protocols (Charters, 2003).

Gibson (1997) suggested an additional tool for the think-aloud process; an actual orientation before a think-aloud interview has begun. An orientation would allow the participants to preview the reasoning and format of think-aloud research to reduce confusion, yet must be carefully done as to not prompt participants to use a particular process. Heeding this advice, I will spent ten minutes giving one example of a non-mathematics related think-aloud task in which I explained how the non-verbal keep-talking prompt will work and the goal of the think-aloud approach (see the Appendix for a full description of the 10-minute pre-interview process in the Structured Interview).

As mentioned earlier, each of the five participants were considered as a “small, tightly focused” descriptive case. This was also suggested of think-aloud participants by Rankin (1988), who suggested the case-study approach to think-aloud participants, although his own analysis of such participants has been primarily quantitative. Such an approach to think-aloud qualitative researchers is suggested to allow the illustration of “the complexities of a situation...show the complexities of human characteristics” and view and describe results as naturally as possible, which is likely most revealing (Charters, 2003). Results from the structured interview were then transcribed and the frequency of SCK use was noted.

Why MKT Tasks and the Think-Aloud Approaches Were Chosen

Olson et al. (1984) states that using think-aloud technique is one of the most effective ways to understand teacher differences in performing the same task. Each teacher completed the same six tasks of varying difficulty (level of difficulty is determined by quantiative analyses by Hill in 2007, discussed further in Chapter 4). MKT tasks hypothesized to fall into the realm of SCK were used as determined by previous factor analysis (Ball, Schilling & Hill, 2007; Hill, 2007).

The cognitive abilities of each participant must also be taken into consideration when choosing tasks. For example, tasks must be grade and content appropriate. Clearly, tasks involving advanced trigonometric ideas often taught in Algebra 2 would not be appropriate as a task for middle school Algebra teachers just as the notion of 1-1 correspondence as taught in first grade would also not be appropriate. Although both fall into the category of Patterns, Functions and Algebra, neither is grade-appropriate. That is, items that previously factored into the category of Patterns, Functions and Algebra at the middle school level were chosen (Ball, Schilling & Hill, 2007; Hill, 2007).

Lastly, although the released items are not taken from the MKT survey directly, they have been used in the past on the MKT survey or are very similar to those that are currently used. In particular, they were used successfully in interview-based reliability and validity efforts yielding a rich understanding and description of specialized content knowledge use (Ball, Schilling & Hill; Hill, 2007; Hill, et al., 2008). I hypothesized that participants would also offer insight into decision-making throughout the structured interview and encouraged all participants to do so. I strongly suspected participants would provide insights into their beliefs as they performed the various SCK tasks.

Section D – The Focused Interview

At this point, participants have taken the beliefs survey, the MKT survey and have participated in the first interview involving think-aloud responses to six MKT tasks. The second interview took place the same day of the first interview, or in two cases within twenty-four hours, since retrospective data is most reliable when the time lag between think-aloud recording and the respective questioning is short (Gibson, 1997). Retrospective questioning was used to illuminate and expand upon the first interview's think-aloud results with the goal of adding depth and understanding regarding information revealed in the first

interview. Also, Qi (1998) suggested that an additional interview with retrospective questioning also allows participants to 'validate' the researchers interpretation of their think-aloud utterances. Gibson (1997) warns, however, that it is better to let participant recall the task as much as possible prior to their using the video-tape as a prompt. Hence, participants were given an opportunity to comment on each of the tasks, only given the task and a prompt to recall as much of their thinking as possible. After this process, participants then used the video-tape as a prompt to recall the task and reasoning.

At the end of the interview, the participants were asked additional questions regarding the nature of their beliefs about mathematics, teacher demographics, school culture, educational background, work and educational background and any personal information that might be relevant to pedagogical content beliefs (for example, were there any experiences that were significant throughout their schooling or other part of their lives that the participant feels is significant in the development of their beliefs about mathematics or beliefs about the teaching and learning of mathematics). The set of questions used in included in the Appendix. Finally, participants were also asked any further clarification questions in regards to their beliefs resulting from the beliefs survey or the first structured interview, particularly if contradictions were noted. All results were transcribed.

Section E – Observations

Beswick (2005) and others (Battista, 1994; Pajares, 1992) argue for the importance of gathering multiple data types both within and outside the classroom because the link between teachers' beliefs and how beliefs influence practice is complex. Allowing participants to share their thoughts in multiple contexts through a methodological lens useful in understanding highly complex situations.

I observed and audio-recorded classroom events over three instructional periods (three math classes) in an effort to learn how mathematics knowledge was being used by teachers and provide a more coherent picture of the interplay between SCK and teachers' beliefs as enacted within the classroom. Field notes were used in addition to audio-recording to add visual details such as teacher/student location, visual context of the classroom and other relevant details as they arose (posterboards, etc.). Observations were made in reference to the following tasks of teaching below (Ball, 2005). Also, any potential inconsistencies in the beliefs-to-practice connection were noted at this time.

Table 6. Ball's (2005) Mathematical Tasks of Teaching

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Observational notes were transcribed, noting each occurrence of SCK use. The following table from Raymond (1997) was used to classify, in terms of category and frequency, a teacher's teaching practices.

TABLE 7. Raymond's (1992) Criteria for the Categorization of Teachers' Mathematical Teaching Practice

Traditional

- The teacher instructs solely from the textbook.

- The teacher follows lesson plans rigidly.
- The teacher approaches mathematics topics in isolation.
- The teacher approaches mathematics instruction in the same pattern daily.
- The teacher has students engage only in individual paper-and-pencil tasks.
- The teacher creates an environment in which students are passive learners.
- The teacher poses questions in search of specific, predetermined responses.
- The teacher allows no student-to-student interactions.
- The teacher evaluates students solely via exams seeking “right answers.”

Primarily Traditional

- The teacher instructs primarily from the textbook with occasional diversions from the text.
- The teacher creates an environment in which students are passive learners, occasionally calling on them to play a more active role.
- The teacher primarily evaluates students through standard quizzes and exams, only occasionally using other means.
- The teacher primarily encourages teacher-directed discourse, only occasionally allowing for student-directed interactions

Mix of Traditional and Socio-constructivist

- The teacher teaches equally from textbook and problem-solving activities.
- The teacher creates a learning environment that at times allows students to be passive learners and at times active explorers
- The teacher evaluates students’ learning equally through standard quizzes and exams and alternative means such as observations and writing.
- The teacher encourages teacher-directed and student-directed discourse.

Primarily socio-constructivist

- The teacher primarily engages students in problem-solving tasks.
- The teacher primarily presents an environment in which students are to be active learners, occasionally having them play a more passive role.
- The teacher primarily evaluates students using means beyond a standard exam.
- The teacher encourages mostly student-directed discourse.

Socio-constructivist

- The teacher solely provides problem-solving tasks.
- The teacher selects tasks based on students’ interests and experiences.
- The teacher selects tasks that stimulate students to make connections.
- The teacher selects tasks that promote communication about mathematics.
- The teacher creates an environment that reflects respect for students’ ideas and structures the time necessary to grapple with ideas and problems.
- The teacher poses questions that engage and challenge students’ thinking.
- The teacher has students clarify and justify their ideas orally and in writing.

- The teacher has students work cooperatively, encouraging communication.
 - The teacher observes and listens to students to assess learning.
-

Reliability and Validity

No matter how well one prepares to deal with the issues surrounding reliability and validity, inevitably, unplanned issues arise during data collection. Each of the five parts of this study carry reliability and validity issues specific to the method, yet there are also overarching themes that are relevant to the study as a whole. In addressing reliability, detailed documentation is an agreed upon strategy that aids in establishing both reliability and validity (Yin, 2003; Creswell, 2009). Hence, every detail of each procedure, including an extensive description of the process and context of the study will be provided. Multiple types of data will be used to support any inferences drawn from both the think-aloud protocols and the analysis of data at large.

In regards to interviews, the interaction between the researcher and participant is “inherent in the nature of interviewing” (Seidman, 2006, p. 22). Hence, issues of reliability and validity require that the researcher acknowledge that the meaning of the data is at least in part, influenced as a result of the researcher and participant interactions (Seidman, 2006). Lincoln and Guba (1985) discuss the issues surrounding reliability and validity from qualitative perspective, using notions of trustworthiness, credibility, transferability, dependability and confirmability, each of which will be addressed specifically in Part II of Chapter 4. It is important to note that due to the nature of qualitative research, reliability is limited.

Specifically relevant to think-aloud protocols, Ericsson and Simon (1980) note that even if their view of thought processes is necessarily incomplete, verbal reports such as those from think-aloud data are a “reliable” source of information about thought processes (p.

247). However, during the process of data collection, probing must be used carefully as to not lead or influence the participant, although some bias is an inevitable result of human interactions. Specific probes and hand signals, as well as specific tasks were designed to minimize bias will be used (Ericsson & Simon, 1980).

The ultimate reliability and validity of Part II of the study lies within Chapter 4, where these issues will be specifically addressed for Sections A – E, but also how the data was analyzed *across the five data sources*. The need for triangulation clearly arises from the ethical need to confirm the validity of the processes in Parts I – V of the study. Specific to the method of case study, this could be done by using multiple sources of data (Yin, 1984). The problem in case studies is to establish meaning rather than location. Construct validity is especially problematic in case study research. In 1994, Yin proposed three remedies to counteract this: using multiple sources of evidence, establishing a chain of evidence, and having a draft case study report reviewed by professional peers.

Reliability, although difficult to attain, can be addressed. One of the most important methods is the development of the case study protocol. Yin (1994) presented the protocol as a major component in asserting the reliability of the case study research. A typical protocol should have the following sections: an overview of the case study project (objectives, issues, topics being investigated), field procedures (credentials and access to sites, sources of information), and case study questions (specific questions that the investigator must keep in mind during data collection). More information regarding how reliability and validity was specifically addressed will be provided in Chapter 4.

Summary

Even after the extensive work of the LMT research group over the years, there is still a continued need to understand *how* teachers use SCK and specifically what factors influence the relationship between a teachers' SCK and actual effective teaching practice due to the inherent complexities involved. The often confusing discrepancy between teachers' level of mathematics knowledge and actual teaching practice must be clarified if we are to further our understanding of quality teaching in general. I continue to build upon the extensive work of the LMT research group in uncovering the specialized content knowledge used in quality mathematics teaching, as well as extend the this work to include the role teachers' belief systems in how this knowledge is used.

CHAPTER 4

DATA ANALYSIS AND FINDINGS

This section encompasses the analysis and findings for Part I of a 2-part multi-method dissertation. In Part I, it was proposed to investigate the factor structure of two subsets of the Mathematics Knowledge for Teaching (MKT) model, vital mathematics knowledge used in quality mathematics teaching, knowledge specifically referred to as common content knowledge (CCK) and specialized content knowledge (SCK) by the Learning Mathematics for Teaching Research Group at the University of Michigan and beyond. Detailed information about this group can be found at:

<http://sitemaker.umich.edu/lmt/people>. CCK and SCK are two subsets of a larger model, the MKT model, developed by the Learning Mathematics for Teaching research group (Hill, Schilling & Ball, 2005).

I began the quest of understanding the content and organization of CCK and SCK by first informally analyzing the MKT Survey and its survey items developed by Hill (2007). I then tested the first factor model, a similar model proposed by Hill (2007) involving tasks aimed at measuring Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK). The observed variables upon which all models were tested resulted from data gathered by Hill in 2005. The data resulted from the administration of the MKT survey on a large, nationally representative set of middle school mathematics teachers (excluding grades 5 and 9). Hill (2007) also performed similar factor analyses, hypothesizing a model that the MKT 2005 survey data tasks were representative of either factor CCK or SCK. She found that both factors were highly correlated, implying a one-factor model versus the

hypothesized two-factor model (Hill, 2007), yet the concept of specialized content knowledge remains an important and distinct one in understanding the distinct types of mathematical knowledge that teachers hold. In Part I of my dissertation, I found that classroom teaching, without the use of SCK in the classroom, was quite different in the level of quality from teachers adept at using such knowledge.

To begin analyses similar to Hill's 2007 work, I proposed a closely related model to Hill's 2007 model, Model 1. I then reorganize the MKT survey tasks under a new model, Model 2, based on the results of the analysis of Model 1. In particular, based on the results of testing Hill's model and the content of the MKT survey items, I change the categorization of fourteen survey items, moving the fourteen items classified as SCK into the category of CCK. Additionally, I reflect upon theoretical considerations of MKT, introducing four new factors underlying CCK and SCK, in an attempt to explain the organization of the underlying observed variables. I then add CCK and SCK as global or second-order factors upon this structure from Model 2 and refer to this second-order model as Model 3. I propose several final modifications based upon the overall results of each of the above steps.

Lastly, a cross-validation of the initial and final models was conducted and the size of the initial sample allowed for such a technique to be used. When modifications are made to an initial model, cross-validation is suggested. Cross-validation is also suggested to evaluate the stability of the model under investigation (Cliff, 1983; Cudeck & Browne, 1983). All models were tested using the statistical software package Mplus (Muthén & Muthén, 2011).

Before proceeding, I provide descriptive statistics relevant to the forthcoming discussion of the various CFA models tested. I present the descriptive statistics for the observed measures and a brief discussion of their meaning, as well as the estimated

correlation matrix of the observed variables in the Appendix. I then present the results of the various model fit indices including various goodness-of-fit statistics and relevant parameter estimates. I conclude by an overall discussion of suggested changes.

Descriptive Statistics

I begin by providing a review of analyses previously done in Chapter 3 and additional descriptive statistics about characteristics of teachers in the sample. According to Hill (2007), the final raw dataset contained four teachers with invalid codes for credential type, so these four data points were considered as missing. Again using the NCES data, Hill (2007) notes that she found no statistically significant differences in school size, number of classroom teachers, school Title 1 eligibility and place (rural vs. urban) when comparing participants with missing data to those without missing data. United States regions showed marginally significant difference in response rates with the Midwestern teachers more likely to respond (68%), and Western teachers less likely to respond (57%). Pupil-teacher ratios were marginally significant, with responders having slightly fewer students on average than nonresponders. Additional observations were missing from each variable, however, and in some cases, a variable's missing data number was significant. Deleting variables was not preferred due to the specialized nature of each task and the limited number of tasks.

However, it is difficult to draw conclusions as to why certain data is missing. Removing observations can, at times, lead to biased results. I initially performed multiple imputation using MIANALYZE in SAS (2011), which basically creates several imputed values for each missing value. Each of the imputed values forms a distinct dataset upon which analysis can be done (King et. al, 2001; Little & Rubin, 2002; Rubin, 1987). In comparing the mean score on individual survey tasks with different results from imputed values, however, there were significant mean differences between the initial dataset and newly created datasets.

Hence, for variables with missing observations, the mean value for respective MKT task were used in place of missing data.

I verified Hill's descriptive statistics on the initial dataset before addressing the issue of missing data (this was included in Chapter 3). For example, with weights applied, the dataset is demographically similar to the U.S. population of middle school teachers, as discussed in Chapter 3. The analytic sample contains 598 middle school teachers, with the following qualifications and descriptors in Table 8 below. This table closely matches Hill's (2007) table for comparison purposes.

TABLE 8. Teacher Characteristics (%)

Teacher Characteristics¹	
Teaching assignments	
Remedial mathematics	9.6 (± 2)
Special education mathematics	7.8 (± 2)
General mathematics	
Integrated mathematics program	57.1 (± 4)
Prealgebra	
Algebra	16.5 (± 3)
Geometry	
Other mathematics classes	48.3 (± 4)
	38.4 (± 4)
	7.6 (± 2)
	9.8 (± 2)
Credentials, grade level	
Any of grades K – 4	
Any of grades 6 – 8	50.5 (± 4)
Any of grades 10 – 12	97.8 (± 1)
	47.2 (± 4)
Teaching Experiences	
Have taught elementary (K – 4)	
Have taught high school (10 – 12)	22.6 (± 3)
Credentials, subject matter	
All subjects	25.4 (± 3)

Mathematics	42.4 (± 4)
Science	71.6 (± 4)
Other subject matter	12.5 (± 3)
	25.0 (± 3)
Credential status	
Fully credentialed	89.7 (± 2)
Preliminary	6.5 (± 2)
Emergency, waiver, or intern	1.8 (± 1)

95% Confidence Intervals

We can see that a bit above one-third of the sample of teachers have taught Algebra.

Almost all teachers hold preliminary or full credentials. However, this is roughly defined, as Hill notes in her 2007 work. Different states define their credentials quite differently, although there is a move toward standardizing these requirements with recent No Child Left Behind efforts. It is interesting to note in Table 3 below that more than 80% of teachers have taken three or more mathematics classes. Additionally, 90% of the teachers took mathematics methods courses and more than one-third took three or more such classes. These results are very similar to Hill's 2007 descriptive results. Further descriptive information is noted regarding participant teacher education below in Table X.

TABLE 9. Teachers' Educational Background (%)

Undergraduate or graduate course work	In mathematics	In mathematics methods
No classes	3.5	11.5
One class	4.7	26.4
Two classes	9.9	24.4
Three or more classes	80.8	35.8
Undergraduate or graduate course work	In mathematics	In mathematics methods
None	14.3	11.9

Less than 6 hours	23.0	22.6
6 to 15 hours	29.7	24.5
16 to 35 hours	18.9	15.6
More than 35 hours	12.1	8.6

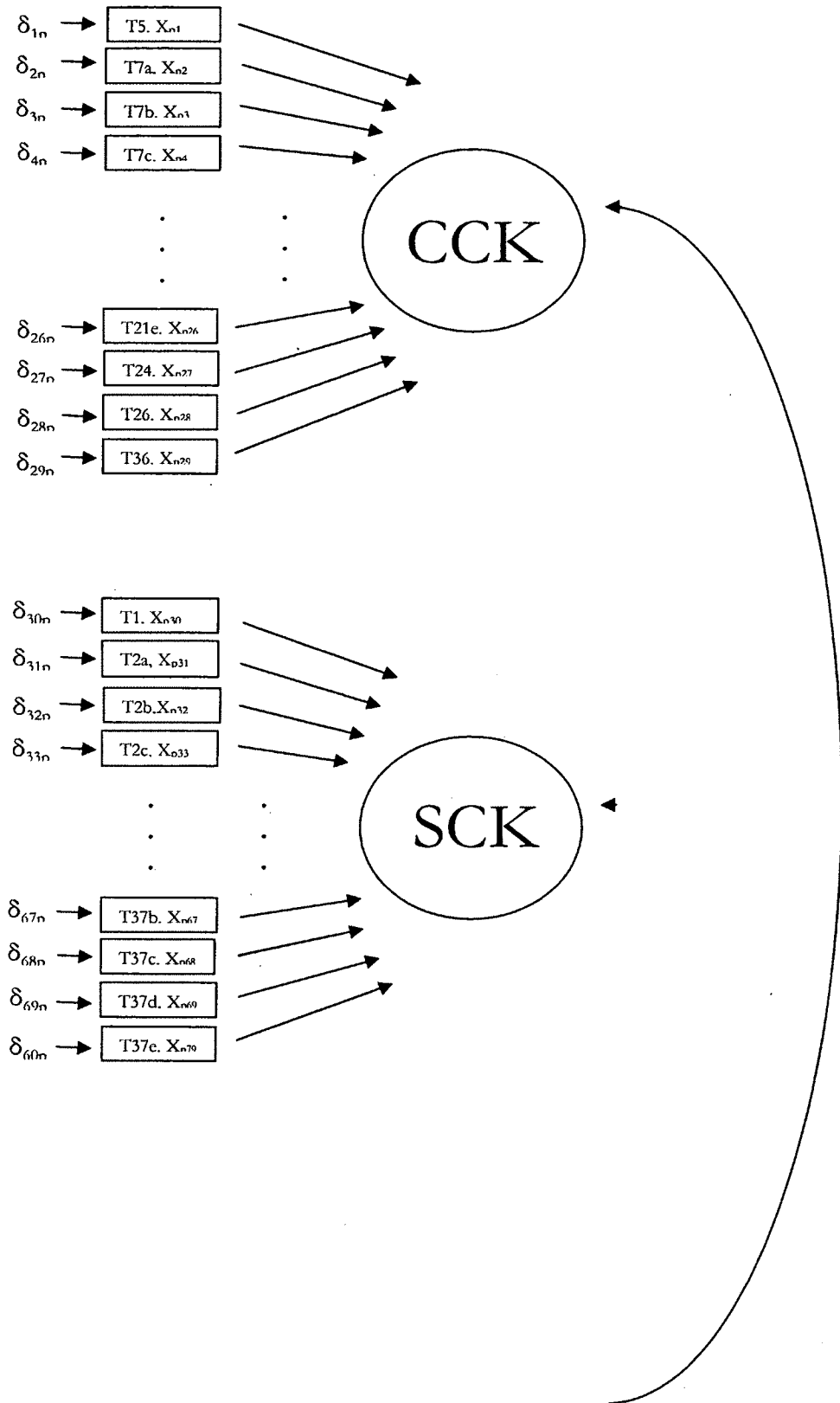
Calibration and validation samples were then selected, both approximately 300 in size, with participants randomly selected using SAS (2011) to constitute both samples.

Descriptive statistics were repeated; to avoid repetition, these statistics will not be repeated here. However, it is important to note that few differences were found when comparing each of the calibration and validation samples to the initial sample (including updates regarding missing data, described above).

Model 1

I begin the steps of testing a series of three distinct confirmatory factor models by first following past suggestions on the organization of survey items from earlier documents created by the Learning Mathematics for Teaching research team that originally created the MKT survey (Hill, Schilling & Ball, 2005), including the work done by Hill (2007). Model 1 is comprised of two factors, CCK and SCK. Scales made from items representing SCK and CCK would not have a significant correlation, ideally, if the constructs are in fact delineable. On the other hand, if the scales created are significantly correlated, this would support Hill's (2007) earlier finding that SCK and CCK items, along with factor analysis results are not distinct but suggest a one-factor model. Model 1 is shown below in an abbreviated form first, followed by each of the factors separately described in more detail.

Figure 5. Model 1 Overview.



It is important to note that the survey items cannot be fully disclosed for several reasons.

One significant reason is that the survey continues to be used regularly across the United States and beyond to assess the mathematical knowledge of teachers. Hence, I will attempt to describe items in each construct for the reader without disclosing details of the items.

Released items are available that are in some cases, similar to the tasks that are not yet released. In cases where released items are similar to specific tasks discussed, released items will be used. Two tasks are described below, with key task examples from each of the two constructs, CCK and SCK. The tasks below are similar, but not identical, to MKT tasks and are used for illustrative purposes only.

MKT Task 8: Mrs. Jones is looking at student responses to a homework problem given the night before to her middle school math students. Students have created statements involving properties of real numbers. Help Mrs. Jones decide if the six statements generated by students below are true or false for the set of real numbers.

MKT Task 10: Mr. Crowley is exploring his middle school curriculum for a story problem that would involve students in identifying non-linear functions. Help Mr. Crowley look through the five example story problems below to choose the story that does not represent the behavior of a non-linear function.

Testing the fit of Model 1

Before fitting Model 1, it is imperative to discuss both theoretical and practical issues that precede the development as well as participating in the testing of an acceptable measurement model. When planning a factor analytic study such as this, it is ideal to have part in the survey development continuing to the development and testing of the model (Hatcher, 1994). However, the development and much of the work involving the theory of the MKT model was well-developed prior to my work here. Hence, much of the survey design

decision making has been done during the creation and testing of the models particular to this dissertation.

Generally speaking, we are looking for evidence that the indicator variables are truly measuring the constructs of interest, in this case, CCK and SCK, and that the measurement model demonstrates an acceptable fit to the MKT 2006 data, i.e., confirming, or not, the underlying factor structure of Model 1. Initially, each latent variable is allowed to correlate freely with the other latent variables in the model. After assessing the model, which is referred to as the confirmatory phase. A next step is the need to discuss the reliability and validity of the model, as well as to conduct modifications to achieve a better fit. The latter is the modification phase, where modifications are made to achieve a better model fit based on sample data, rather than theory (or prior research). Hence, the latter needs to be confirmed with different sample. For this reason, I have created and used both a calibration and validation sample in the overall process (each sample resulted from a random split of the initial data set in SAS). The factor validity for Models 1, 2 and 3 were all tested on the calibration sample (for $n=298$). The second sample ($n=299$) then served as the validation sample for the initial and final models proposed.

In the above Model 1, the manifest (indicator) variables, represented by the rectangular boxes, are hypothesized to provide a subjective measure (the self-reported responses to the MKT survey tasks) of either the latent factor CCK or SCK as denoted above. Hatcher (1994) explains that each of the δ_i 's associated with the manifest variables are disturbance terms that can be large for a particular construct, such as CCK for example, if other constructs that have important effects on CCK have been excluded from the model. That is, a disturbance term represents "causal effects on the endogenous variables due to such

things as omitted causal variables, random shocks, and misspecifications of equations” (James, Mulaik & Brett, 1982, p. 163).

There are additional issues to consider as necessary conditions for performing CFA. Some conditions were clearly satisfied upon inspection of the data and sample size, such as a minimal number of three indicator values (in some readings, the suggestion is at least four or five) and a minimal number of observations. In each of the models, the minimum number of indicator values was nine, and the number of observations was close to 300. The minimum number of observations in each of the subsamples was met (Hatcher, 1994).

The distribution of data is also a consideration. Outliers and non-normal distributions (large splits in the case of dichotomous variables) can cause problems with analysis, particularly influencing the chi-square test, one of several criteria used to assess the overall fit of a model. This particular dataset (and hence its subsets) did not contain variables with large splits. Lastly, certain statistical packages were not equipped with the statistical tools necessary to deal with non-linear relationships and indicator variables that are not continuous. Since the survey data responses were coded to 1 for a correct survey response and 0 otherwise, a software package capable of handling discrete variables was necessary, hence the use of Mplus (Muthén & Muthén, 2011).

It is important to note that survey responses were ultimately scored and responses classified as “correct” (the value in this case is 1) or “incorrect” if any of the other incorrect responses were chosen. That is, variables were ultimately dichotomous. This was the technique used by Hill (2007) and hence for consistency, this approach was also used here.

I now return to a discussion of Model 1. The table below describes the organization of the MKT tasks for Model 1.

TABLE 10. Organization of Tasks Model 1.

SCK	CCK
1	5
2	7a-7d
3a-3d	8a-8f
4	9
5	11a-11c
6	12
10	13
16	
17	14
18a-18d	15
20	19a-19d
22a-22d	21a-21e
23	24
25	26
27	36
28	
29	
30a-30c	
31	
32	
33	
34	
35	
37a-37e	

Note that Model 1 indicates that CCK and SCK are correlated as indicated by the double arrows in the above model. I expected this correlation to be high, considering Hill's original 2007 findings. There are thirty-seven observed variables as indicated by the thirty-seven rectangles representing the MKT survey items. Observed variables in the CCK column of Table X load on the factor CCK and the observed variables in column SCK of Table X load on factor SCK. Residuals associated with each observed variable are uncorrelated. Specifically, each item-pair measure has a nonzero loading on the factor it was designed to measure and a zero loading on the other factor.

In discussing results of each model, I will first discuss the assessment of the model as a whole via goodness-of-fits statistics, followed by the evaluation of individual parameter

estimates. For Model 1, the following goodness-of-fit statistics are shown in Table X below. It is *crucial* to note that although the goodness-of-fit statistics listed here differ in how they are computed than the identically-named tests used in instances where the data are continuous. For more on this matter, see the formula given in the *Mplus* “Technical Appendices” at <http://www.stat-model.com>, under “chi-squaredifference testing.”

TABLE 11. Model 1: Goodness of Fit Statistics

Model 1: Goodness of fit statistics	
Chi-Square Test of Model Fit	Model 1
Value	648.24
Degrees of freedom	82.9
<i>p</i> -value	0.0001
Chi-Square Test of Model fit (Baseline Model)	
Value	1798.52
Degrees of freedom	224
<i>p</i> -value	0.0001
CFI/TLI	
CFI	.734
TLI	.758
Loglikelihood	
H ₀ value	-6473.47
H ₁ value	-6646.88
Information Criteria	
Number of free parameters	142
AIC	11652.34
BIC	11886.36
Sample-size adjusted BIC	11992.74
RMSEA Estimate	
Estimate	0.176
90% confidence interval (CI)	0.172 0.178
Probability RMSEA ≤ .05	0.76
WRMR	
Value	1.085

The Chi-Square Test of Model fit for Model 1 is 648.24. Formally speaking, the null hypothesis proposes that the factor loadings, factor variances and covariances, and residual

variances, to be discussed in detail shortly, are credible (Byrne, 2012). With a Chi-Square (χ^2) Test of Model fit of 648.24, with approximately 82.9 degrees of freedom and a probability of less than .0001, this suggests that the overall fit of the model is not likely to be adequate. However, the Chi-Square test of Model Fit is not conclusive, given the current data. For example, this statistic is sensitive to sample size, among other reasons. When assessing a model, there are several criteria used for model assessment (Byrne, 2012; Hu & Bentler, 1999) and I will process through the other criteria to further assess the overall fit of Model 1.

The next statistic discussed is the χ^2 value for the baseline model. Relative fit indices, such as the χ^2 value in this case, compare the chi-square value for the hypothesized Model 1 to the baseline model, a model in which all of the variables are uncorrelated. Generally speaking, the baseline model has a large χ^2 value, indicating a poor fit. Relative fit indices include the normed fit index and comparative fit index. Comparing the χ^2 value for the hypothesized model with the baseline model assesses the extent to which the hypothesized model fits the data, as compared to the baseline model. The hypothesized model had a χ^2 value of 648.24, with 82.9 degrees of freedom compared to 1798.52 with 224 degrees of freedom for the baseline model. The extent to which the χ^2 value for the hypothesized model is less than the χ^2 value of the baseline model, is considered an improvement in model fit (Byrne, 2012), which is clearly the case.

I now turn to two incremental indices regarding model fit, CFI (Bentler, 1990) and TLI (Tucker & Lewis, 1973), which again compare the baseline fit with the hypothesized model fit. Values closer to 1, and in particular, higher than .95, are considered an indication of a better model fit (Hu & Bentler, 1999). For this model, the CFI and TLI are .73 and .76, respectively, again indicating that Model 1 is likely not the best fit. It is important to note

that a significant amount of time has passed since Hill's (2007) work and hence the specific fit statistics are not available as a further reference.

Both the AIC (Akaike, 1987) and BIC (Raftery, 1993; Schwartz, 1978) are considered possible predictive and parsimony-corrected categories of fit indices. According to Byrne (2012) using the AIC and BIC requires the *loglikelihood* values also shown above in Table X.

In contrast to the CFI and TLI, both of which focus on comparison of *nested* models, the AIC and BIC are used in the comparison of two or more *nonnested* models, with the smallest value overall representing the best fit of the hypothesized model. Both take into account model fit (as per the χ^2 value) as well as the complexity of the model (as per model degrees of freedom or number of estimated parameters). The BIC, however, assigns a greater penalty in this regard and, thus, is more apt to select parsimonious models (Arbuckle, 2007).

Absolute indices of fit, the RMSEA and SRMR, do not rely on a comparison with a reference model, i.e. these values decrease with fit as goodness-of-fit improves, obtaining values close to zero when the model fits perfectly (Browne et al., 2002). The routine use of RMSEA is highly recommended (MacCallum and Austin, 2000) for a variety of reasons, one of which is the ability to build confidence intervals around the RMSEA value. The RMSEA value is 0.176. We can be 76% confident that the true RMSEA value in the population will fall between 0.172 and 0.178. Yu (2002) suggests the use of the Weighted Root Mean Square Residual (WRMR) when using categorical (in this case, discrete) data versus the Standardized Root Mean Square Residual (SRMR). Yu (2002) suggests a cutoff value of 0.95 can be considered to indicate a good fit for a model. Thus, these results leave some room for doubt that Model 1 is an ideal model fit.

The above results suggest that the current model, categorizing knowledge as either CCK or SCK may not be an ideal fit. However, considering overall model fit is only one aspect of determining the fit of a model. I now turn to specific parameter estimates that not only shed light on specificities of model fit, but also can inform changes that might lead to model changes for fit improvement. I will begin by reviewing significance tests for factor loadings. Technically speaking, a nonsignificant factor loading can mean that the indicator variable involved is not doing the best job of measuring the underlying factor (this indicator variable could be considered for reassignment or removal). It is also important to note that the factor loadings are computed *Mplus* differently for discrete variables (standardized estimates are used). There were no problematic standard errors (near-zero values) that might indicate an estimation problem. Additionally, all factor loadings were significant at $p < .001$. There were twelve indicator variables that had corresponding loading values that were small and statistically significant (.60 and below). Before proceeding, I revisit Task 10 below.

MKT Task 10. Mr. Crowley is exploring his middle school curriculum for a story problem that would involve students in identifying non-linear functions. Help Mr. Crowley look through the five example story problems below to choose the story that does not represent the behavior of a non-linear function

Task 10 demonstrated a large negative normalized residual with the other indicator variables assigned to SCK. However, Task 10 demonstrated a large positive residual with the tasks associated with CCK. Other tasks with this same issue are included in this list of twelve tasks: 3, 5, 10, 14, 16, 17, 18, 25, 27, 30, 35, 37. Analogously, tasks in this list showed a large negative normalized residual with the other indicator variables assigned to SCK.

However, the tasks in this list demonstrated a large positive residual with the tasks associated with CCK. Hatcher (1994) suggests that tasks such as Task 10 may be assigned to the wrong factor, i.e. Task 10 (as well as the other 11 tasks) could be considered to be reassigned to construct CCK.

Consider what the above task is asking a teacher to do and what specific knowledge a teacher might be using in this case. The teacher is not deciding whether or not the story problems are appropriate for a unit on linear functions (an SCK task). The teacher is not deciding the best story problem for a middle school student audience (an SCK task).

Although the contents of the actual task cannot be revealed, the task required only for the teacher to solve each of the five story problem examples, using primarily procedural knowledge. In other words, Task 10 appeared to be accessing a teacher's common content knowledge instead. Recall the definition of specialized content knowledge (Ball, Hill & Schilling, 2005).

Specialized content knowledge (SCK), is mathematical knowledge beyond that expected of any well-educated adult but not yet requiring knowledge of students or knowledge of teaching. Many of the common tasks of teaching require significant mathematical resources, but do not yet necessarily require knowing about students or teaching... Like pedagogical content knowledge it is closely related to practice, but unlike pedagogical content knowledge it does not require additional knowledge of students or teaching. It is distinctly mathematical knowledge, but is not necessarily mathematical knowledge familiar to mathematicians.

I argue that the knowledge accessed in Task 10 is not the best indicator of knowledge specifically unique to teachers; I further argue that mathematicians hold this knowledge and

that this knowledge is not solely specialized content knowledge. I also argue that the knowledge accessed is primarily procedural, but this will be expanded upon as the discussion continues. To further illustrate the point, consider the following description of a task similar to a MKT survey item, Task 3 below.

Task 3. Ms. Ellen asked her students to explain why the number 1 is not prime. Help Ms. Ellen inspect the four choices and decide the best explanation.

This, and other similar tasks, are asking teachers for common content knowledge; specifically knowing, conceptually, why a value fits a particular mathematical definition or not. Other mathematically literate (ML) adults such as mathematicians or those working in professions where mathematics is central also hold this information. If you asked the average adult, this person may or may not have this knowledge, but the knowledge comes directly from the concept and its derivatives itself, not specialized knowledge that teachers hold that is purely mathematical in nature. Compare this example with the following example below.

Task 31. Mrs. Staples is the following proportion equivalencies. Her students have not yet learned the procedure or concepts behind cross-multiplication. Considering her students' background in this sense, which of the four proportion equivalencies below would be hardest to solve?

In this case, unlike in the above examples, I would argue that Mrs. Staples is using specialized content knowledge. A teacher answering this question is looking at a variety of proportion problems, with a variety of possible solutions and must consider the types of

knowledge that middle school students hold at a specific point in the mathematics curriculum and decide which problem is accessible to them, *specifically the mathematics involved*. This knowledge is fundamentally different than CCK.

After recategorizing these 14 items from SCK to CCK, and reviewing all MKT survey items, I noticed a substructure under both the CCK and SCK constructs. Specifically, CCK items fell into two seemingly distinct categories: procedural knowledge (knowing how, mathematically) and conceptual knowledge (knowing why, mathematically). (It is important to note that procedural and conceptual knowledge are accessed simultaneously as well). SCK items also fell into two categories: choosing mathematical scenarios (CMS) and evaluating student claims (ESC), **yet it is important to note that I am not referring to teaching tasks, but rather, I am referring to the specialized, purely mathematical, content knowledge inherent within such tasks.**

Thus, I proposed a new model for the mathematics knowledge of middle school mathematics teachers with the survey tasks falling within a structure slightly different in nature, yet closely related, to the initial two-factor model proposed. The proposed model, Model 2, is shown below in Figure X. This model was tested first to determine if Model 2 was in fact a better fit than Model 1. Model 2 is shown below. The delineation between CCK and SCK is still vital and clear with CCK encompassing categories PK and CK and SCK encompassing CMS and ESC. After testing Model 2, Model 3 will be tested, which will include all previously discussed modifications as well as the addition of the global factors, CCK and SCK.

I first proceed with describing tasks in the four new categories below:

PK Task 21. Mr. Nadeau is looking at a textbook problem he plans to give his students.

Help Mr. Nadeau decide which of the following four solutions is a correct solution to the following equation involving one variable.

CK Task 3. Ms. Ellen asked her students to explain why the number 1 is not prime. Help Ms. Ellen inspect the four choices and decide the best explanation.

CMS Task 31. Mrs. Staples is the following proportion equivalencies. Her students have not yet learned the procedure or concepts behind cross-multiplication. Considering her students' background in this sense, which of the four proportion equivalencies below would be hardest to solve?

ESC Task 34. Mrs. Doubtfire is reviewing class work while walking around her classroom and working with students. A group of 4 students has four different methods for finding 30% of any positive real number, x . Help Mrs. Doubtfire look at the four student responses and determine if she can tell the student if their method will work for any positive real number.

Testing the Fit for Model 2

I now proceed with the following goodness-of-fit statistics for Model 2, shown in Table X below, along with the results from Model 1. The task organization is listed first.

Table 12. Task Organization of Model 2. (SCK and CCK are here for descriptive purposes but not yet added as global constructs).

SCK	
CMS	ESC
2	1
4	23
20	24
22a	28
22b	32
22c	33
22d	34
29	
31	

CCK	
PK	CK
5	3a
7a-7d	3b
8a-8f	3c
9	3d
10	6
11a-11c	13
12	14
15	16
18a-18d	17
19a-19d	25
21a-21e	27
26	
30a-30c	
35	
36	
37a-37e	

Figure 6. Model 2

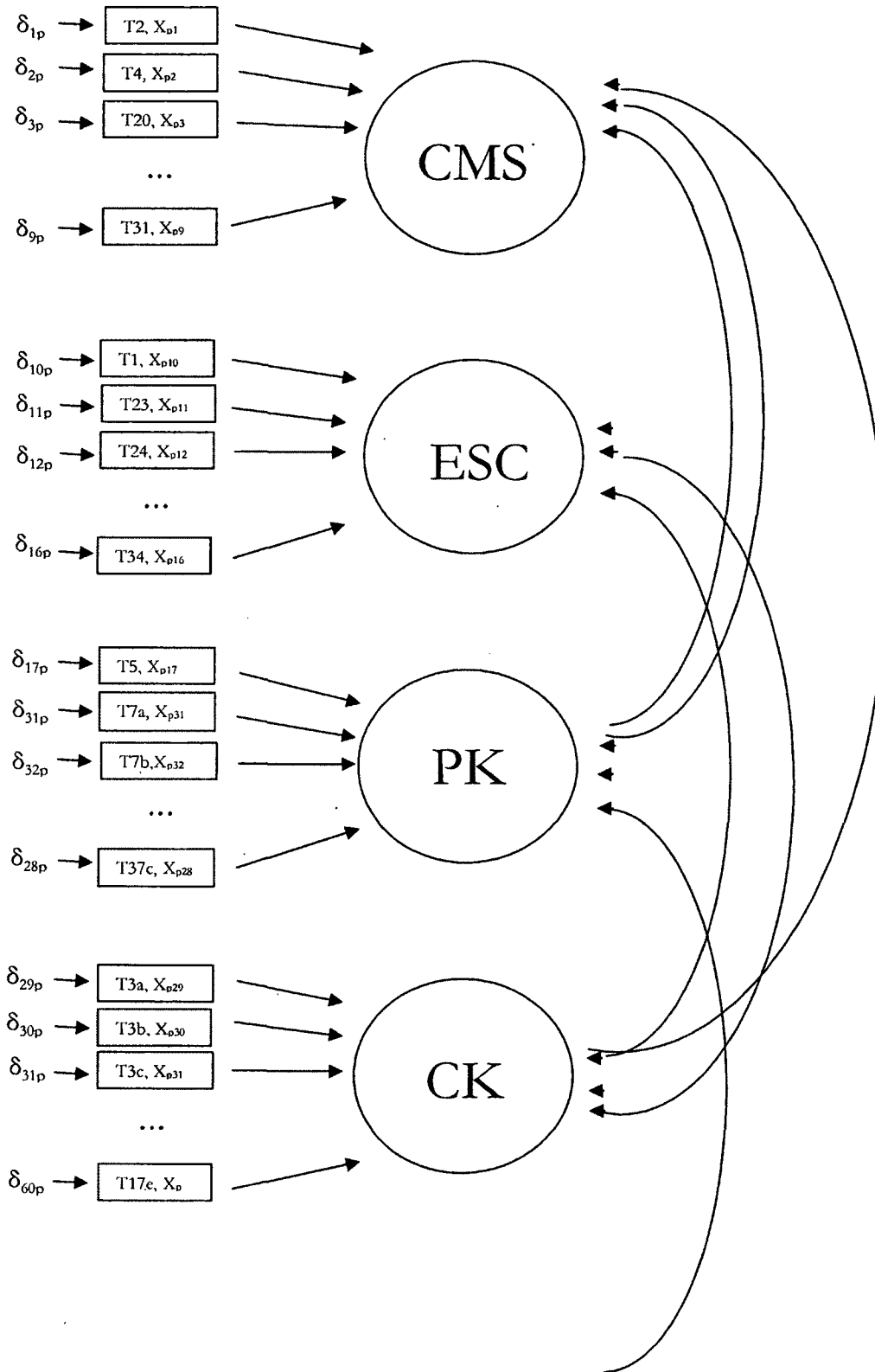


TABLE 13. Model 1, 2: Goodness of Fit Statistics

Model 1, 2: Goodness of fit statistics		
Chi-Square Test of Model Fit	Model 1	Model 2
Value	648.24	448.82
Degrees of freedom	82.9	211
p-value	0.0001	0.0001
Chi-Square Test of Model fit (Baseline Model)		
Value	1808.55	1698.52
Degrees of freedom	224	282
p-value\	0.0000	0.0001
CFI/TLI		
CFI	.734	.91
TLI	.758	.92
Loglikelihood		
H ₀ value	-6473.47	-5873.58
H ₁ value	-6646.88	-5468.24
Information Criteria		
Number of free parameters	142	139
AIC	11652.34	12388.34
BIC	11886.36	12188.59
Sample-size adjusted BIC	11992.74	12295.90
RMSEA Estimate		
Estimate	0.176	0.033
90% confidence interval (CI)	0.172 0.178	0.030 0.035
Probability RMSEA <= .05	0.76	0.82
WRMR		
Value	1.085	0.98

Table X. Overall model 1 and 2 goodness-of-fit statistics

With a Chi-Square (χ^2) Test of Model fit of 448.82, with approximately 211 degrees of freedom and a probability of less than .0001, this suggests that the overall fit of the model is not likely to be adequate. However, there has been a substantial drop in the Chi-Square test of Model Fit from Model 1 as well as an increase in both the CFI and TLI values. The RMSEA value is 0.033, we can be 82% confident that the true RMSEA value in the population will fall between 0.030 and 0.035. This result leaves some room for doubt that

Model 2 is an ideal model fit, although certainly indicates a possible improvement in fit over Model 1.

I now turn to specific parameter estimates. As with the previous model, parameter estimates were reviewed for appropriateness. All parameter estimates were checked to have the appropriate sign (+/-) and relative size. Additionally, as with Model 1, the covariance matrices were positive definite, with no correlations greater than one and no negative variances.

Standard errors were investigated first. Values that approach zero will not be defined and values that are significantly large denote parameter estimates that cannot be determined (Byrne, 2012). There is no specific criteria for *extremely large* and hence standard errors were inspected as to their relation to other values. Extreme differences were not present. Nonsignificant parameters (factor loadings), in a large sample such as this, should be deleted. There were two such nonsignificant parameters, namely PK by 35 and CK by 17. Both were deleted. Otherwise, all unstandardized estimates were reasonable and statistically significant.

Standardized factor loadings were reviewed next. The factor loadings for CMS ranged from .40 to .94 with two values less than .60. The factor loadings for ESC range from .48 to .83, with three values less than .60. The factor loadings for PK range from .49 to .94 with seven values less than .60, and lastly the factor loadings for CK range from .38 to .91, with four values less than .60. Hence, the majority of factor loadings were at least moderately large, with all but two, namesly PK by 35 and CK by 17, statistically significant.

I review the normalized residual matrix next. I followed the suggestion to review the normalized residual matrix (versus the residual matrix) with the expectation that the unit of measurement varies from indicator variable to indicator variable due to the complex nature

and content of the survey tasks (Hatcher, 1994). Of the listed largest residuals in the Mplus output, several were noteworthy (see table below).

TABLE 14. Residuals

Path	Residual
20 and 33	-3.582
31 and 33	-2.149
10 and 16	-2.917
15 and 16	-2.749
12 and 17	-2.019
26 and 17	-2.008

It is important to note that task 20 and 31 were hypothesized to measure CMS, whereas 33 was predicted to measure ESC. Additionally, tasks 10, 15 and 12 were predicted to measure PK, whereas tasks 16 and 17 both were predicated to measure CK. Looking back at the original covariance matrix values for the above tasks (in comparison to the predicted covariance values), it is clear that Model 2 underpredicts the strength of the relationship between the above noted tasks. It is possible also that tasks 20 and 31 were influenced by task 33 which affects 20 and 31. One might consider reassigning 33 to CMS; yet this would not be appropriate because the standardized coefficient from 33 to ESC was .88 and statistically significant. A similar situation exists involving tasks 10 and 15. Tasks 12 and 26, however, were not doing a great job of measuring measuring PK, indicated by their low standardized coefficient from task 12 to PK and task 26 to PK. Additionally, tasks 12 and 26 have large negative normalized residuals with other indicator variables predicted to measure CK. Hence, tasks 12 and 26 were reassigned to CK and their associated paths

removed from PK. This is theoretically interesting because, when solving mathematical tasks, it is common to access both procedural and conceptual knowledge. The following task illustrates this dilemma:

CK and PK Task 8. Mrs. Graham is looking at a textbook problem and is planning to do the problem before assigning it to students. The problem requires that she evaluate algebraic expressions and then decide what subsets of the real numbers (which includes the real number set itself) the answer belongs to.

Testing the fit of Model 3

I now move to testing the fit for Model 3, which includes the reassignment of tasks 12 and 26 to CK that took place before testing Model 2 and all previously mentioned modifications. The additional difference with Model 3 is that the model is comprised of a second-order factor structure, with an addition of the two global factors, CCK and SCK. As discussed above, the constructs CCK and SCK have theoretical importance, and serve as an overarching organization quite apparent in practice and evident in past research. An overview of Model 3 is shown below. The overall goodness of fit values for Model 3 follow, showing a small amount of improvement when adding SCK and CCK as global factors.

FIGURE 7.. Model 3

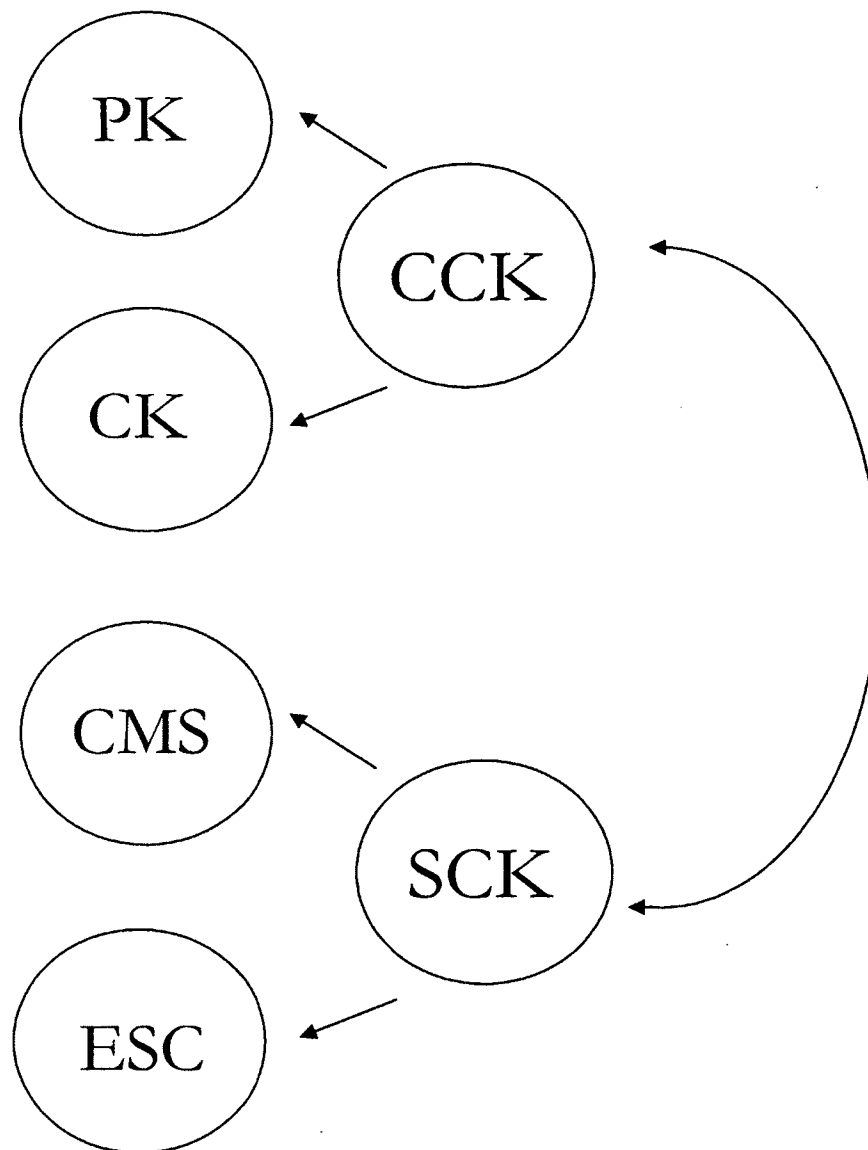


TABLE 15. Models 1, 2, 3: Goodness of Fit Statistics: Sample 1

Model 1, 2, 3: Goodness of fit statistics: Sample 1			
Chi-Square Test of Model Fit	Model 1	Model 2	Model 3
Value	648.24	448.82	381.68
Degrees of freedom	82.9	211	218
<i>p</i> -value	0.0001	0.0001	0.0001
Chi-Square Test of Model fit (Baseline Model)			
Value	1808.55	1698.52	1726.37
Degrees of freedom	224	282	291
<i>p</i> -value	0.0000	0.0001	0.0001
CFI/TLI			
CFI	.734	.91	.92
TLI	.758	.92	.94
Loglikelihood			
H ₀ value	-6473.47	-5873.58	-5837.02
H ₁ value	-6646.88	-5468.24	-5479.38
Information Criteria			
Number of free parameters	142	139	136
AIC	11652.34	12388.34	12128.28
BIC	11886.36	12188.59	12118.39
Sample-size adjusted BIC	11992.74	12295.90	12124.10
RMSEA Estimate			
Estimate	0.176	0.033	0.034
90% confidence interval (CI)	0.172 0.178	0.030 0.035	0.030 0.035
Probability RMSEA ≤ .05	0.76	0.82	0.82
SRMR			
Value	1.085	0.98	0.97

With the use of CFA on a higher-order structure comes additional alterations to the analyses done previously with the first-order structures. Technically speaking, the factor-loading paths are fixed to 1.0 to avoid the default values from *Mplus* and an equality constraint must be added with respect to the two global factors. This can be handled in

Mplus directly within the input file within the model specifications. We can now discuss the output results for Model 3.

A review of the model fit statistics (CFI = .92; TLI = .94; RMSEA = 0.034) suggests that hypothesized second-order Model 3 exhibits a slightly better fit than Model 2 to the data, currently the calibration sample (n=298). There is a slight improvement in the Chi-Square Tests of Model Fit, with the p-value for such statistics still roughly equivalent to the p-values for Models 1 and 2. The Weighted Root Mean Square Residual, again a preferred statistic over the Standardized Root Mean Square Residual (SRMR), moves slightly closer to the preferred .95 cutoff value. I now turn specifically to unstandardized parameter estimates, noting first that a review of both the first and second order factor loadings are statistically significant.

Due to constraints in space and avoiding repetition, only relevant information is presented here. I inspected the residual variances with respect to the first order factors. Inspection of these loadings is between individual tasks and the first order factors is, not surprisingly, notably different from loadings discussed as in Model 2.

In short, second-order CFA, otherwise known as hierarchical CFA, allows the investigation of whether or not the second-order factors explain correlation among the first-order factors. As mentioned earlier, the interpretation of factor-loading estimates for discrete variables is based on the squared standardized factor loadings (Byrne, 2012). For example, observed variable X has an R^2 estimate of .69 means that 69% of its variance can be explained by the construct of Y to which it is linked. In other words, a correlation among first order factors CMS and ESC (and specifically neither of the other second-order factors, PK and CK) would suggest that SCK is a reasonable choice as a global factor for the constructs CMS and ESC. Similarly, a correlation among first order factors PK and CK (and

specifically neither of the other second-order factors, CMS and ESC) would suggest that CCK is a reasonable choice as a global factor for the constructs PK and CK. Although Model 3 seemingly provides the best overall fit after careful analysis of both overall goodness-of-fit measures and specific parameter estimates, it is not clear that SCK is a global construct providing the best explanation for CMS and ESC. When investigating loadings related to the second-order factor CCK (i.e. PK and CK), CCK appears to be a reasonable choice as a global factor for PK and CK. This is not surprising due to the complexity in identifying, describing and measuring SCK, versus CCK.

Cross-validation

All three models were cross-validated using the second subsample of the original data, sample 2 (n=299). It has been strongly suggested that multiple covariance models be cross-validated given multiple modifications in such a process (Schreiber, 2006). It is imperative to note that this discussion is an overview. The discussion of specific details and the next steps of invariant analyses were left for future work, with this work focusing on the theoretical issues that arose when closely inspecting the differences in the content of MKT tasks.

First, models 1, 2 and 3 were fit to the second subsample of data, sample 2 (n=299). This was possible because the sufficient initial sample size allowed for such a technique to be done. This process tested only the appropriateness of the specific factor pattern as the parameter estimates were free to vary in sample 2. The relevant fit statistics are given here below. It is important to note that there are several other approaches to cross-validation that would strengthen the below discussion; yet due to time restrictions and the theoretical issues related to the content of the tasks, this was set aside for future work.

Table X. Cross-validation of Models 1, 2, and 3 with Sample 2.

TABLE 16. Models 1, 2, 3: Goodness of Fit Statistics: Sample 2

Model 1, 2, 3: Goodness of fit statistics: Sample 2			
Chi-Square Test of Model Fit	Model 1	Model 2	Model 3
Value	548.24	528.62	393.85
Degrees of freedom	98	202	229
<i>p</i> -value	0.0001	0.0001	0.0001
Chi-Square Test of Model fit (Baseline Model)			
Value	1738.55	1698.52	1726.37
Degrees of freedom	220	289	290
<i>p</i> -value	0.0000	0.0001	0.0001
CFI/TLI			
CFI	.78	.90	.91
TLI	.73	.90	.92
Loglikelihood			
H ₀ value	-5423.43	-5798.20	-5804.32
H ₁ value	-5442.38	-5392.88	-5349.29
Information Criteria			
Number of free parameters	142	139	139
AIC	11252.44	12086.66	12028.29
BIC	11680.98	12088.29	12016.02
Sample-size adjusted BIC	11992.73	12122.24	12120.14
RMSEA Estimate			
Estimate	0.172	0.037	0.036
90% confidence interval (CI)	0.170 0.180	0.030 0.040	0.030 0.040
Probability RMSEA ≤ .05	0.72	0.79	0.80
SRMR			
Value	1.094	0.963	0.965

The first model clearly failed to fit the data from Sample 2. Unlike earlier findings, the fit of Model 2 to Sample 2 does not show as much improvement. Model 3, however, demonstrates similar findings when we evaluate the fit of Model 3 for Sample 1. Further work could be done to specifically inspect the item residuals for each model and compare to

further account for the variation between models. Again, more specific analyses such as these were left for future work.

Summary

I began this work with an interest in mathematics knowledge use in the classroom. I am extremely thankful to Heather Hill, who provided me with the opportunity to investigate this area of research via a large, nationally representative dataset. The dataset resulted from her work over the years of 2005-2007 and the administering of the MKT middle school survey with tasks representative of the mathematics teachers use in their everyday teaching. A tremendous amount of work and mathematics education expertise went into the discovery of mathematics knowledge for teaching model (MKT), as well as the development and piloting of the survey tasks. Their work has certainly played a major role in understanding and describing how teachers use mathematics knowledge in the classroom.. I am sincerely thankful for the opportunity to further investigate MKT.

I first note several general limitations before proceeding. Firstly, this is my first experience with the MKT model and research behind the MKT framework. This is also my first experience with both confirmatory factor analysis and the Mplus software. Additionally, there are limits to my own knowledge of Hill's 2007 work simply because I was not involved in that work; hence I was careful here to limit my reporting to my own experience and knowledge available via the various reports from the LMT research group over the years. Since I cannot release information regarding the task contents as these tasks continue to be used in valuable ways both in research and professional development involving the use of MKT and teachers, I did the best I could to describe tasks to help the reader understand the theoretical underpinnings of my findings. This limitation may have caused frustration for

readers at times but was necessary for the protection of future work related to the MKT survey. Task examples I developed as substitute tasks were closely aligned with the design purpose of the MKT survey task.

There were many differences between my work and Hill's 2007 work. I used a different software product than had been used previously; this alone can change (albiet minimally) specific findings. I did not participate in the factor analyses previously performed on this data and hence I do not have relevant values for comparison. Additionally, although I tested an initial model quite similar to Hill's 2007 model, I then reevaluated the organization of the survey tasks, ultimately arguing for a different, yet related MKT model of middle school mathematics teacher knoweldge. When considering the comparision of models, particularly between Model 1 and Model 2, several changes were made and hence I cannot truly state what differences led to the overall improvement of model fit.

The analyses in this chapter could primaliy be improved by having hands-on knowledge of the variety of phases that precede and constitute confirmatory factor analysis (often preceded by exploatory factor analysis especially in the case of a newer survey tool). Such first-hand knowledge would include an exploratory phase, specifically with the reorganized model to provide further understanding of the theoretical underpinnings that ultimately influence the fit of the model. This knowledge would also include specific survey design justifications including the choice of assigning the survey results of each task to either correct or incorrect (a discrete variable) versus a categorical variable based on the categorical response to the task. Detailed validity and reliability would be re-evaluated for the survey, including the specific understanding how the different designs of questions influence overall results.

Model validation is vital in model testing. The sample size in this study allowed for initial validation efforts. Hill (2007) took great care in creating a large enough sample that was useful in this work in replicating the model results with more than one sample. The size was such that I could meet the suggested goal of splitting the data, at random, and estimate each model twice, then comparing the results (Pohlmann, 2004). This procedure specifically allows one to evaluate the stability of the parameter estimates. Overall goodness of fit measures changed significantly for Model 1 and Model 2 when tested to fit the second of two samples. This calls into question the stability of parameter estimates in Model 1 and Model 2. This was not the case for Model 3; however, to truly draw conclusions about the stability of parameters in Model 3, further invariant analyses are needed. These tasks were left for future work, yet the findings here suggest that further understanding is needed in the development and testing of SCK MKT middle school survey tasks.

Nevertheless, there are several significant findings well worth discussing and expanding upon in the future. The key findings of Part I of the study involved new understandings of the measurement characteristics of MKT tasks and overall organization of the MKT model. Ideally, survey tasks that are hypothesized to measure SCK should be accessing the SCK teachers hold. SCK should be clearly defined, theoretically, and survey items should be tested extensively within a series of procedures assessing the validity and reliability of the survey and eventually the theoretical model to be tested. This is quite a challenging task when the measures are designed to indicate such complex constructs such as CCK and SCK. There were middle school MKT survey tasks that were likely designed to measure SCK but primarily access teachers' common content knowledge. One could argue that different teachers might approach tasks differently and that there is no clear determination on the types of knowledge a survey question will actually access. However, the fourteen items

reclassified from SCK to CCK were very similar to other survey tasks I used in Part II of my study in the structured interview; in this setting, the tasks proposed to measure SCK appeared to access only CCK in this setting as well. This will be discussed further in Part II of the study. These findings clearly indicate there is further work to be done in the understanding of the organization of MKT, particularly surrounding the role of SCK.

There was slight improvement in the Chi-Square Tests of Model 3 fit, with the p-value for such statistics still equivalent to the p-values for Models 1 and 2. Model 2, although unstable when tested on Sample 2, demonstrated an improvement in overall fit from Model 1 both in overall goodness of fit statistics and upon the inspection of specific parameter estimates, with few exceptions. The major change from Model 1 to Model 2 was two-fold; the movement of twelve SCK items to the CCK construct as well as renaming two subconstructs for both SCK and CCK. This was decided based on the analysis of Model 1 and the reconsideration of survey items that were initially classified as SCK but upon further inspection, appeared to be accessing knowledge that more closely aligned with the definition of CCK. I now turn to the theoretical discussion of this particular issue. Consider again the two tasks below, similar to survey items 10 and 31. These two items illustrate the core theoretical issues at hand.

PK Task 10. Mr. Crowley is exploring his middle school curriculum for a story problem that would involve students in identifying non-linear functions. Help Mr. Crowley look through the five example story problems below to choose the story that does not represent the behavior of a non-linear function.

CMS Task 31. Mrs. Staples is the following proportion equivalencies. Her students have not yet learned the procedure or concepts behind cross-multiplication. Considering her students' background in this sense, which of the four proportion equivalencies below would be hardest to solve?

In the first task, classified initially as SCK, a teacher is likely accessing knowledge falling within the definition of CCK. The five story problems in the actual survey require the teacher to read through each problem and do the problem to determine whether the story problem represents a linear or non-linear function. The teacher is clearly accessing procedural knowledge but is also likely accessing conceptual knowledge related to the concepts of function, linear function and non-linear function. The classification and investigation of conceptual and procedural knowledge is not new to mathematics education. In fact, these ideas have been the subject of a number of research articles and books (Hiebert, 1983). These ideas have also been the heart of current debates of which type of knowledge should be the focus of the mathematics classroom.

The focus of the problem, however, is procedural due to the nature of the five stories; although the reader does not have access to the stories, the stories are consistent with the middle-school level and are simple in nature; the functions presented clearly fall into either linear or non-linear and are typical functions at the middle school level. When I first came upon a task like this, it was clear that there were going to be concerns with a model that classified this task as SCK. These tasks (there were twelve such tasks) were reclassified as PK, with an eventual added hierarchical category of CCK. Tasks such as these that primarily accessed conceptual knowledge were classified as CK, still with an overarching category of CCK.

Along the same lines, task 31 is clearly accessing knowledge that is different in nature than CCK. A teacher answering task 31 is thinking about several things, one of which is her knowledge of middle school student thinking and access issues surrounding the specific new concept of cross-multiplication. When do middle school students best learn the concept of cross-multiplication problems? What background information must I ensure students hold to be able to teach this new concept? The teacher is also likely accessing other information that typically informs teachers in their decision making process, accessing knowledge well outside of the realm of CCK and SCK as well.

The main idea, however, is that the nature of tasks 10 and 31 are essentially *very* different and clearly illustrate the important findings of the LMT Research group over the years. Theoretically, the difference is clear; we have yet to discuss the difference between CCK and SCK, as used in the practice of teaching, which is discussed next in the Analysis and Findings Chapter involving the second yet related study in Part II of the dissertation. Looking ahead, this key difference sets the stage for the discussions in the analysis, findings and interpretation of Part II of the study. Ultimately, through the observation of teachers in practice, key differences between CCK and SCK emerged, shedding further light onto the organization of MKT.

II: Understanding How Teachers' Beliefs are Manifested in SCK Use

How beliefs influence instructional practice and the use of specialized content knowledge is not yet clear. The complications begin with researchers trying to access teachers' ever-changing belief systems. Further complicating matters is that teachers are not always aware of the beliefs they hold and are not always able to articulate the beliefs they *are* aware of (Beswick, 1996). Moreover, teachers may hold beliefs that are not enacted in practice. For

example, a teacher that believes mathematics should be taught from a socio-constructivist perspective may not be able to enact these beliefs for a variety of reasons; one reason might be the teacher's perception of a lack of support of his or her beliefs. The literature is rife with evidence that there are internal and external factors mediating the beliefs-to-practice teaching connection (Pajares, 1992).

Most agree that teacher beliefs act as a filter through which teachers make instructional decisions and that understanding how this filter works is key in understanding the beliefs-to-practice connection (Clark & Peterson, 1986; Handal & Herrington, 1993). More recently, experts in this area have strongly recommended that clarifying the beliefs-to-practice connection requires a close look at beliefs within the teacher's specific discipline, focusing on how the particular discipline's knowledge is used. It is also strongly suggested that teacher beliefs be investigated within multiple contexts using multiple methods. This suggested approach allows teachers to indicate and expand upon their self-reported beliefs, clarifying their perspectives in multiple contexts.

An overview of the complete 5-method data collection process is shown below in Figure X and described in detail in the next section.

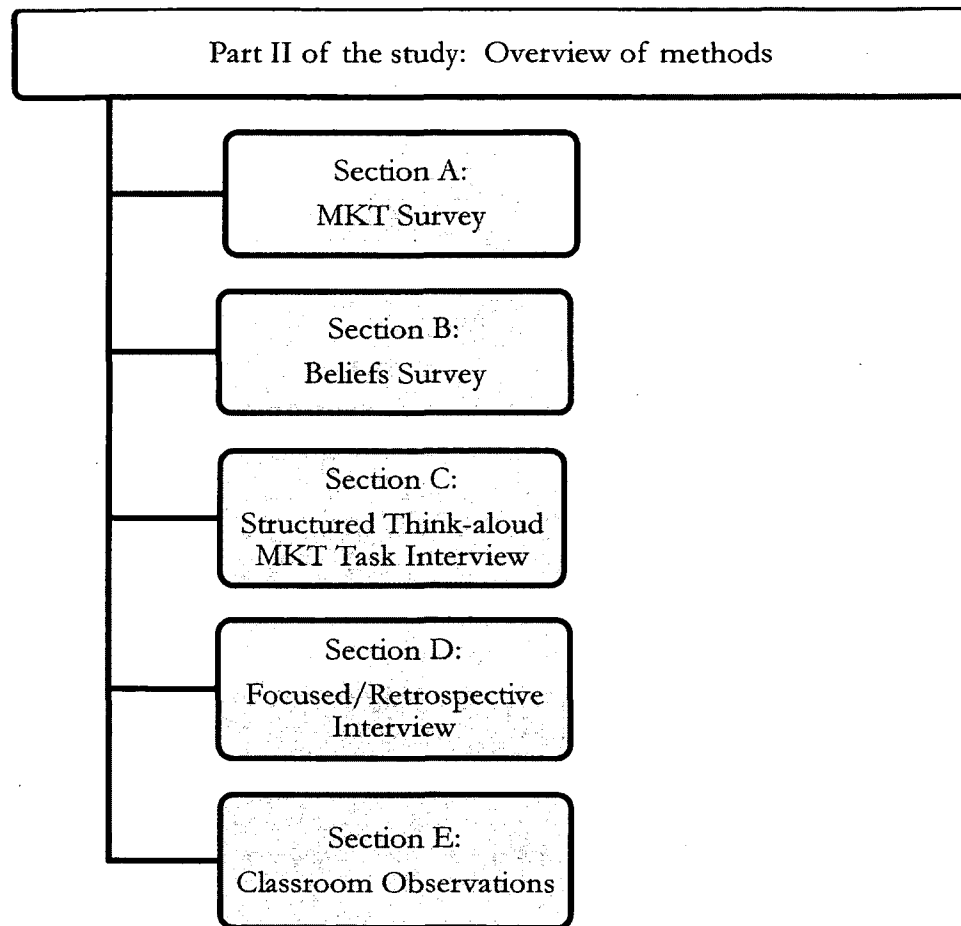


FIGURE 8. Overview of the Data Dollection Methods in Part II of the Study.

Methods Overview

There are five subsections in which five data sources are gathered, denoted Sections A – E in the following discussion. In Section A of the study, teachers participated in an approximately one-hour, on-line multiple choice self-report survey, the Mathematical Knowledge for Teaching (MKT) Instrument developed by the Learning Mathematics for Teaching Project team (Hill, Schilling & Ball, 2004), tasks encompassing the mathematical knowledge used in the practice of teaching, including both common content knowledge

(CCK) and specialized content knowledge (SCK). The tasks specifically fell into three content areas (3 sub-sections of the survey): middle school numbers and operations (20 questions), and both elementary and middle school patterns, functions and algebra (17 questions each). Administering this survey served several purposes, one of which was to expose participants to the MKT practice-based mathematical tasks encompassing specialized content knowledge, in part to prepare teachers for the structured interview where teachers talking through similar tasks in more depth in Section C of the study, discussed in more detail below.

An additional purpose was to gauge participant levels of MKT and in particular, participant levels of CCK and SCK. MKT scores allowed for group comparisons in MKT performance to national results; participant survey results are grouped accordingly into low, medium and high MKT scores. Additionally, performance on the two middle school sections (numbers and operations, patterns functions and algebra) were linked to 2005-2006 national results, with the caveat that teachers completed multiple forms. The choice of the use of multiple forms was primarily due to the grade span of participants and willingness of participants to commit to a longer MKT survey to provide a broader span of participant knowledge for the researcher. The elementary form, if used with a middle school participant pool, could result in a ceiling effect; however, scores on this section are high enough to cause such a problem.

Participants did not have access to MKT scores during the study and committed to certain test-taking procedures including a quiet, non-interrupted time period and an optimal time of day for test taking. Although given the opportunity under certain circumstances to retake the survey, no participant indicated this was necessary. Although participants were asked, no participants reported problems while taking the survey.

The MKT is not a criterion-referenced test, and hence I am not making claims about whether a teacher has a sufficient level of MKT. Instead, IRT scores were presented (IRT provides a test of item equivalence across groups) on a scale of -3 to +3 (increments of 1), where -3 represents exceptionally low MKT and +3 represents exceptionally high MKT. It is important to note that the MKT survey was designed such that the average teacher would answer the average item correctly, 50% of the time (Hill, 2007). Hence, a theoretical average MKT score is denoted by 0. Each point on the scale roughly corresponds to a teacher answering 12 to 14% more items correctly than the previous point on the scale. A teacher at +2 on the scale responded correctly to approximately 40% more survey items correctly than a teacher at -1 on the scale, for example.

In Section B of the data collection process, teachers participated in a paper-based, 45-question beliefs survey (each question based upon a 5-point Likert scale) focused on gauging teachers' beliefs about mathematics and teachers' beliefs about mathematics teaching and student learning (Peterson, et al, 1989). It is important to note that the focus of this study, however, is on teachers' beliefs about mathematics teaching. First used in a 1989 study involving the effects of teachers' pedagogical content beliefs in mathematics on teaching instruction, this survey has since provided valuable feedback regarding how certain aspects of teachers' belief systems have impacted mathematics instruction (Ball, 1990; Raymond, 1997; Stipek, et al., 2001).

Gathering multiple sources of data regarding teachers' beliefs generally clarified the beliefs to practice connection often shown to be unclear in research where beliefs are investigated within only 1-2 contexts (Fang, 1996; Beswick, 2005). Teachers were encouraged to make as many notes as they would like on the beliefs survey sheet and to ask for clarification. Teachers did not ask for clarification but all participating teachers did add

clarifying comments on their beliefs surveys. Teacher beliefs indicated by participants on the survey were readdressed during the focused interview, Section D of the study.

In Section C of the study, teachers participated in a video-recorded, structured think-aloud interview developed by the author, where the participant engages with and talks through MKT tasks including the use of specialized content knowledge. Teachers noted a variety of theoretical instructional decisions and were allowed the possibility of providing their perception of beliefs behind their decisions while working through tasks. Next, in Section D, teachers participated in a focused interview developed by the author where participants observed the video from Section C and provided retrospective clarifying comments, as well as educational history information. Questions asked were aimed at clarifying the characteristics and complexities of the teachers' beliefs system (see Appendix, X). Two example questions are shown here below:

- What are your beliefs about the subject mathematics? Does mathematics change?
What words immediately come to mind?
- How would you describe your beliefs about mathematics teaching?
- How do students best learn mathematics?
- How does your current school climate influence your beliefs? Your teaching?

In the last phase of the data collection process, Section E, I observed each participant in the classroom for three one-class periods within a one-week period (with the exception of one participant whose 3 observations were done over three class period in one day). The choice to observe for three classes was suggested to observe overall quality of instruction after a review of multiple teacher quality observation tools (Hill, 2005). The length of each class varied per school, ranging from fifty minutes to one hour and twenty minutes.

Observations included comparable content involving patterns, functions and algebra and

numbers and operations, although it was not the goal to compare knowledge use among participants within a specific content area. During observations, *at least one* of the following tasks of teaching (Ball, 2005) constituted a teaching episode during which teachers were observed using specialized content knowledge. That is, at least one mathematical task below including specialize content knowledge had to be present to constitute a unit of analysis.

TABLE 17. Ball's (2005) Mathematical Tasks of Teaching

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematiactl content of textbooks
Modfying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Follow-up questions, as an extension of the focused interview, were completed as needed for clarification purposes, particularly focusing on clarifying the beliefs-to-practice connection. Each of the study participants completed the five-part study in a short time interval; approximately one week's time-period to gauge consistency of beliefs, recall, and other factors influencing the reliability and validity of the study. One participant fell outside of the one-week range due to a life-changing circumstance.

Study Participants

This multiple-case study consisted of a total of 10 cases. Each of the ten participants were recruited via an state-wide mathematics teaching organization's e-mail list, the New Hampshire Teachers of Mathematics (NHTM). Twelve participants responded and eleven were recruited. One was unable to finish the study; the remaining ten constituted the ten case studies for this inquiry. All ten teachers were currently middle school teachers, teaching at grades levels 6 - 8, although one teacher taught a multi-level class, grades 5/6. Seven teachers taught in public school; three taught in the same private school. Three teachers in total taught grade 6 (Jake, Robert and David), four teachers taught grade 7 (Julie, Nancy, Sarah and Melissa) and three teachers taught grade 8 (Anna, Debra and Beth).

Of the ten teachers, eight teachers were state-certified within the same state to teach middle school, with the exception of Nancy and Julie. Four teachers were state-certified within the same state to teach at the middle *and* secondary level (Jake, Anna, Beth and Debra). One teacher, Jake, was a National Board for Professional Teaching Standards (NBTPS) certified teacher in mathematics. Three of the teachers who were certified at both the middle and secondary level had formal experience in teaching high school math. Anna taught Algebra 1 at the high school level for 2 years. Debra had the most high school teaching experience, at 15 years. Beth had the most middle school teaching experience, with a total of 9 years. Although Jake initially indicated no formal high school teaching experience, upon further investigation, Jake noted that his one-semester internship was at the high school level. Jake relayed that his cooperating teacher allowed him to teach class regularly during his internship, at a large public high school, which included Algebra 1, Algebra 2 and Geometry classes. Jake's full-year internship was similar, at the high school level.

Preliminary Analyses: Five Data Sources

Ultimately, although five data sources were present *and* analyzed separately, the data sources fell into three overarching categories, which served as an organizational forum for the five data sources. For example, although the beliefs survey served as a primary indicator of teacher belief categories, the focused interview, although a separate data source, ultimately served as a forum for clarification and expansion upon the beliefs survey. In other words, these two data collection tools were the primary informers of the classifications of teacher beliefs. Pragmatically speaking, it was more useful to view these two data sources as one set of information on teacher beliefs, one ultimately informing the clarity of the other. Also, the focused interview specifically served as added reliability to the beliefs survey.

Similarly, the MKT survey, structured MKT task interview and classroom observations served as closely related data sources, with both ultimately informing the findings involving specialized content knowledge. For example, the data gathered from the MKT survey and MKT survey tasks interview added further insights and credibility to SCK observed in the classroom. Surprisingly, Part I played a major role in setting the stage for understandings that were primarily revealed during teacher observations. The specifics of this general discussion follow.

Analysis of MKT Survey Data

Each of the study participants took the MKT survey which included tasks representative of middle school mathematics knowledge for teaching, including specialized content knowledge. The Mathematical Knowledge for Teaching (MKT) Instrument was developed by the Learning Mathematics for Teaching Project (Hill, Schilling & Ball, 2004). The MKT tasks were designed to measure the mathematical knowledge used in the practice of teaching,

encompassing the content areas of middle school numbers and operations (20 questions), and both elementary and middle school patterns, functions and algebra (17 questions each).

Participant MKT scores allowed for group comparisons in MKT performance; participant survey results are grouped into low, medium and high MKT scores. Additionally, performance on the two middle school sections (numbers and operations, patterns functions and algebra) were linked to 2005-2006 national results, with the caveat that teachers completed multiple forms, discussed in detail above. Results are shown in Figures X, Y and Z below, separately for each of the three MKT forms. Figure X denotes results from the middle school patterns, functions and algebra survey. Note that the scores ranged from -2 to 2, but the total *possible* score of a participant can range from -3 to 3, with -3 representing exceptionally high levels of MKT and 3 representing exceptionally low levels of MKT.

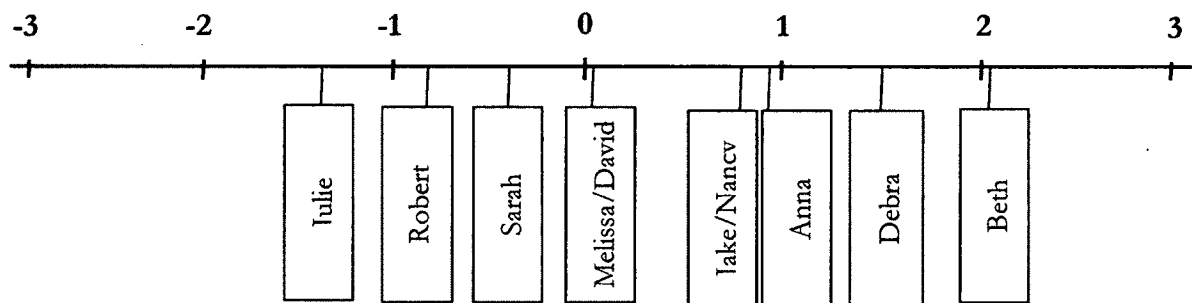


FIGURE 9. Above, all case study participants ordered by IRT scores on the middle school Patterns, Functions and Algebra MKT survey, ranging from -3 to 3.

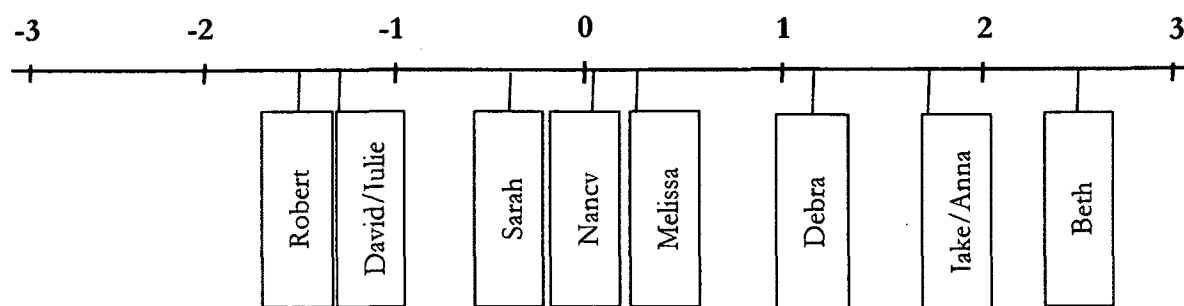


FIGURE 10. Above, all case study participants ordered by IRT scores on the middle school Numbers and Operations MKT survey, ranging from -3 to 3 .

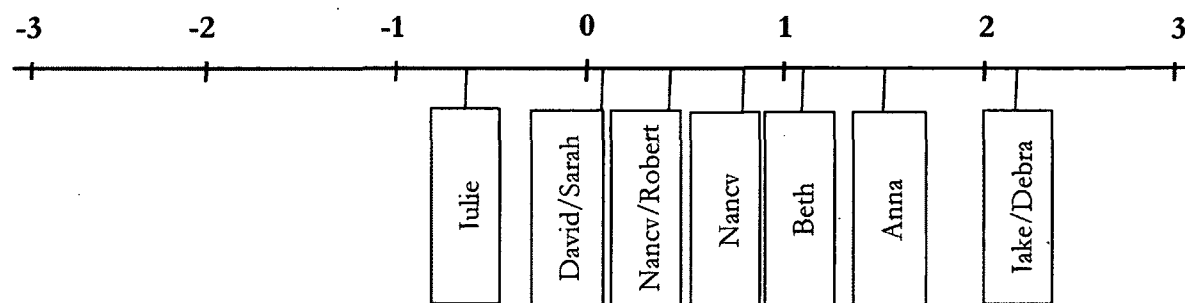


FIGURE 11. Above, all case study participants ordered by IRT scores on the *elementary* school Patterns, Functions and Algebra MKT survey, ranging from -3 to 3 .

Low, middle and high categories were previously determined in Hill's 2007 work. It is interesting to note that three teachers that scored in the group labeled high on one the Patterns, Functions and Algebra portion of the MKT survey scored the highest on each portion of the survey in the three distinct areas of MKT content. These three teachers were also the only teachers with both middle and high school teaching experience and dual certification at both levels. This aligns with Hill's (2007) finding that middle school teachers that also had high school teaching experience performed better than those with middle school experience alone. In 2007, Hill found that teachers with more math courses, a math-specific certification and high school teaching experience tended to have higher levels of MKT. These three teachers similarly have the greatest number of math courses. Compared

with the other participants, these three teachers are also unique in that they completed a teacher education program with a full one-year internship in a mathematics classroom.

Teachers varied in their performance for each of the three content areas. For example, although Beth scored in the high range for both middle school content surveys, her score on the elementary school version of patterns, functions and algebra was lower than her other scores. Most notably, when responses were inspected by CCK versus SCK, all ten teachers scored lower on SCK items than CCK items. Hill's (2007) study supports a similar finding. Common content knowledge items were found to be significantly easier than specialized content knowledge items. In particular, Hill's (2007) findings suggest middle school mathematics teachers have a "strong grasp of the mathematics required of the children they teach – knowledge of number systems, procedures, basic ratio/proportionality, and algebraic problem solving. Less firmly grasped, however, are items intended to represent elements of specialized content knowledge" (Hill, 2007, p. 104).

In this same study, Hill also found that explanations behind mathematical ideas and procedures had the highest difficult levels in this population of middle school math teachers, a significant component of the specialized content knowledge necessary for quality mathematics teaching. Amongst the ten participants, items of a similar class (explanations behind ideas and procedures) also were the lowest scoring items.

A review of the literature reveals that a good number of beliefs mathematics teachers hold generally fall into the three categories: beliefs about the nature of mathematics (beliefs about mathematics), beliefs about teaching mathematics, and beliefs about learning mathematics. It has been fruitful in the past to use these three categories to serve as a framework in which to categorize teachers' beliefs (Ernest, 1989a; Thompson, 1991). The

focus of this particular study is on teacher beliefs about teaching mathematics, but the other two categories are closely related and therefore, at times, were discussed.

A teacher's beliefs about the nature of mathematics is "his or her belief system concerning the nature of mathematics as a whole (Ernest, 1989). Beliefs about the content of mathematics have been shown to impact instructional practices (Raymond, 1991). Beliefs about the nature of mathematics shown to influence views about teaching and learning include:

1. Instrumentalist view (mathematics as an unrelated , fixed, accumulation of facts, rules and skills),
2. Platonist view of mathematics (mathematics as a static but unified body of discovered knowledge, both fixed and dynamic, both absolute and relative)
3. Problem-solving view (mathematics as an ever-changing, relative, doubtful, expanding field created as a cultural product) (Thompson, 1973; Ernest, 1989).

Two belief *systems* that dominate the literature in mathematics education as influencing the teaching and learning of mathematics, often used to categorize teacher beliefs in terms of the teaching and learning of mathematics are *traditional* and *nontraditional or socio-constructivist* (sometimes referred to as progressive or reform-based) (Cobb, 1990). Through much of the literature, the terms instrumentalist and traditional hold essentially the same meaning; similarly with the terms socio-constructivist and problem-solving, although the platonist category shares some characteristics with the socio-constructivist category.

Socio-constructivist teaching and learning of mathematics generally involves situations in which students are able to learn mathematics by individually and socially constructing mathematical knowledge (Handal, 2003). Instructional strategies typically associated with this socio-constructivist teaching and learning emphasize problem-solving and generative

learning, as well as reflection and exploration. Traditional instructional methods are generally associated with a behaviorist perspective on education and include an emphasis on the transmission of knowledge, formulas, procedures and drill, i.e. emphasizing products rather than the process (Beswick, 2005). Such methods have been shown to encourage isolated learning, as well as leading to students conforming to the teacher's established methods of problem-solving (Wood, Cobb & Yackel, 1994). Although there are variations of these two categories, these are the two main systems typically identified and discussed in current research (Ernest, 1991).

There is disagreement as to *how* each view of teaching and learning of mathematics (socio-constructivist versus traditional) influences student learning but there is general consensus that much of the socio-constructivist teaching strategies are preferred due to their tendency to produce quality mathematics teaching. Lee Stiff, a past President of the National Council for Teachers of Mathematics prefers to use the term “standards-based mathematics” versus *constructivist*.

“Reform-minded teachers pose problems and encourage students to think deeply about possible solutions. They promote making connections to other ideas within mathematics and other disciplines. They ask students to furnish proof or explanations for their work. They use different representations of mathematical ideas to foster students' greater understanding. The teacher asks students to explain the mathematics...Sometimes they work with other students, sometimes they work alone...”

In the below beliefs summary model, table X, the first row of beliefs are most closely aligned with a socio-constructivist view of mathematics and mathematics teaching and learning (Beswick, 2005). In Beswick's 2005 work, teachers who held beliefs in the three

categories of the *third* row below had classroom teaching instructional practices typical of a socio-construcivist perspective of teaching and learning (Taylor et al., 1993). Similarly, teachers who held beliefs in the *first* row had classroom teaching practices that would be described as methods typical of traditional classroom practice. Beswick refers to the middle category row as encompassing the middle ground of beliefs between the two other rows.

TABLE 18. An expansion of Beswick's Relationships Between Beliefs (Beswick, 2005).

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989)
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of discrete knowledge, emphasis of performance
Platonist	Content-focused with an emphasis on understanding	Active construction of content understanding
Problem-solving	Learner-focused	Autonomous exploration own interests, social construction of knowledge

In understanding the various beliefs that teachers hold about teaching mathematics, I used Beswick's 2005 framework as a theoretical bases upon which to categorize beliefs that resulted from both the beliefs survey, focused interview and if applicable, the structured interview. It is important to note that I did not expect beliefs to *only* fall within these categories. Although the beliefs survey and focused interview did ask questions that related to these categories, there were many opportunities for teachers to comment on beliefs without an imposed conceptual framework. Despite this, teachers typically placed their

beliefs within the frameworks in rows 1 and 3; the categorization teachers beliefs clearly fell as a dichotomy between row 1 or row 3.

The comprehensive criteria for the categorization of teachers' beliefs about teaching mathematics is showed in Table X below and is based upon Beswick's work. The same criteria is used in Ann Raymond's comprehensive 1997 study entitled *Inconsistency between a Beginning Elementary School Teacher's Mathematical Beliefs and Teaching Practice*.

TABLE 19. Criteria for the Categorization for Teachers' Beliefs about Mathematics Teaching

Traditional

- The teacher's role is to lecture and to dispense mathematical knowledge.
- The teacher's role is to assign individual seatwork.
- The teacher seeks "right answers" and is not concerned with explanations.
- The teacher approaches mathematical topics individually, a day at a time.
- The teacher emphasizes mastery and memorization of skills and facts.
- The teacher instructs solely from the textbook.
- Lessons are planned and implemented explicitly without deviation.
- The teacher assesses students solely through standard quizzes and exams.
- Lessons and activities follow the same pattern daily.

Primary traditional

- The teacher primarily dispenses knowledge.
- The teacher primarily values right answers over process.
- The teacher emphasizes memorization over understanding.
- The teacher primarily (but not exclusively) teaches from the textbook.
- The teacher includes a limited number of opportunities for problem-solving.

Mix of traditional and socio-constructivist

- The teacher includes a variety of mathematical tasks in lessons.
- The teacher equally values product and process.
- The teacher equally emphasizes memorization and understanding.
- The teacher spends equal time as a dispenser of knowledge and as a facilitator.
- Lesson plans are followed explicitly at times and flexibly at others.
- The teacher has students work in groups and individually in equal amounts.
- The teacher uses textbook and problem-solving activities equally.
- The teacher helps students both enjoy mathematics and see it as useful.

Primarily nontraditional

- The teacher primarily facilitates and guides, with little lecturing.
- The teacher values process somewhat over product.

- The teacher emphasizes understanding over memorization.
- The teacher makes problem solving an integral part of class.
- The teacher uses the textbook in a limited way.

Nontraditional

- The teacher's role is to guide learning and pose challenging questions.
- The teacher's role is to promote knowledge sharing.
- The teacher clearly values process over product.
- The teacher does not follow the textbook when teaching.
- The teacher provides only problem-solving, manipulative-driven activities.
- The teacher does not plan explicit, inflexible lessons.
- The teacher has students work in cooperative groups at all times.
- The teacher promotes students' autonomy.

The belief survey served as an initial indicator only; this tool was useful in starting conversations about beliefs. Although teachers were initially categorized by their responses to the beliefs survey, if later contradictions or evidence arose that contradicted the results to the beliefs survey, the focused interview was used to clarify beliefs and provide a final categorization of teachers' beliefs. Belief categories were final after the focused interview; the goal was for the category to reflect the *teacher's* reflection of his or her beliefs, not the view of the researcher. Hence, belief categories were not altered during or after teacher observations. Specifically, each teacher was put in a category for each of the nine belief categories in the above table only if the majority of the teacher's frequency of indicated beliefs fell into that category. The specific criteria for categorizing teachers' beliefs into the above nine categories was similar to a subset of Raymond's (1997) categories, that align with the constructs in Beswick's 2005, with slightly different terminology. The categories of traditional and nontraditional were not used; rather, categories of primarily traditional and primarily socio-constructivist were used due to the results of this particular study; teachers in this study were not absolute in their categorization. Rather, teachers indicated beliefs that

were either primarily traditional (majority of the beliefs were traditional) or primarily socio-constructivist (majority of the beliefs were socio-constructivist).

It was not possible to clarify all belief inconsistencies that arose via the beliefs survey and focused interview, nor was this a goal of this study to reach a final conclusion about the consistency of a set of beliefs due to the ever-changing and complexity of teachers' beliefs. Steps were taken, however, to note internal and external factors, such as the influence of time on the recollection of events. When inconsistencies occurred, they were documented and included in the analysis. The complexities only enhanced the validity of the study. In determining the nature of the beliefs-to-practice connection, the frequency and description of inconsistencies between the final participant's belief categorization and teaching practice were noted (teaching practice categorization is discussed in detail below).

If a teacher indicated two beliefs on the survey that were distinct and inconsistent, or more than one belief that were inconsistent, the focused interview was used to gain clarification into such issues, if possible. The focused interview provided a forum for the clarification process used when inconsistencies arose. It is important to note that, during analysis and categorization of teacher beliefs, beliefs about teaching were categorized separately from beliefs about learning. Although this initial separation was important in noting potential differences in understanding the overall categories of teacher beliefs, beliefs about teaching and learning overlapped more often than not despite the focus on beliefs about teaching.

The below example of Nancy illustrates the process of clarifying an apparent inconsistency in reported beliefs. Nancy's beliefs about how students learn mathematics, in row 2, are not consistent. In the focused interview, Nancy was able to clarify her perception of her belief in this area. In particular, she reflected upon her beliefs survey response and

indicated that the description of the belief she expressed in her focused interview most accurately represented her current belief about how students learn mathematics best.

TABLE 20. Selected indications of Nancy's belief system.

Belief Category	Nancy's Beliefs Survey	Focused Interview
Beliefs about the nature of mathematics	Mathematics, as a discipline, is a relatively static field.	Math as a subject, there is little change. At this level, K – 12...there is not a lot of change in the curriculum, but changes as to how we teach it.
Beliefs about how students learn mathematics	Students learn best when they follow the teacher's instruction. Students learn best when forming their own connections.	Students really learn the best when they can make their own connections, that's what helped me. I needed to construct ideas for myself.
Beliefs about mathematics teaching	In school, math should be taught in such a way that students can discover their own connections.	I wish I could focus on this more. This is doesn't generally work with kids. Kids really need structure and practice.

All data regarding teachers' beliefs were initially coded by Raymond's above categories and subject to cluster analysis as described by Beswick (2005). Cluster analysis, in the most formal sense, can be described as "dividing objects into groups (clustering) and assigning particular objects to these groups (classification)...clusters are potential classes and cluster analysis is the study of techniques for automatically finding classes" (Lee, 1984). In other words, cluster analysis attempts to uncover the structure in the data by grouping participants according to similarity amongst their responses. The initial analysis began by randomly separating the sample into two equal groups. Groups were analyzed separately and compared results to determine the consistency of emergent clusters across the two subgroups. Proximity was calculated as the Squared Euclidean proximity function with a centroid mean.

The specific cluster analysis procedure used was similar to Beswick's (2005) procedure, i.e. hierarchical and agglomerative. This is essentially a descriptive technique that is helpful in understanding the underlying data as long as specific requirements are met. This approach uses several clustering techniques that produce a hierarchical clustering by starting with each participant as an individual cluster. The procedure then combines the two most similar clusters from this step into a single cluster. Ward's (1969) method was used, considered an effective means of uncovering underlying data structure. This method measures proximity between two clusters in terms of the increase in the SSE that results from the merging of the two clusters, minimizing the sum of the squared distances of points from their cluster centroid values. During this proximity analysis, identifying stages in which significant changes in the agglomeration coefficient occur is a predefined means of stopping the hierarchical agglomeration procedure. The final cluster solutions were characterized by a significant correspondence with the belief categories in Figure X within indicators denoted in Raymond's Table X, beliefs about mathematics teaching. Results are shown below.

Table 21. Summary of the Beliefs About Teaching Mathematics Cluster Categorizations

Cluster Characteristics	Primarily traditional	Primarily Mixed	Primarily socio-constructivist
Anna		X	
Chris		X	
Beth		X	
Sarah			X
Nancy			X
Debra	X		
Robert			X
David			X
Nancy	X		
Julie	X		

Analyzing the Use of Specialized Content Knowledge

Capturing and analyzing specialized content knowledge was just as complex. As to be expected, specialized content knowledge was closely tied to the teaching task as well as common content knowledge. Describing specialized content knowledge use required providing enough context to truly capture the SCK knowledge use. Hence, instead of attempting to isolate SCK, the unit of analysis was a teaching episode that contained a specific teaching task. A list of such tasks is given above in Figure X here. During observations, *at least one* of the following tasks of teaching (Ball, 2005) constituted a teaching episode during which teachers were observed using specialized content knowledge. That is, at least one mathematical task below had to be present for a teaching episode to occur for later data analysis.

TABLE 22. Ball's (2005) Mathematical Tasks of Teaching

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Describing SCK use required allowing teacher ample time and opportunity to observe its use. The biggest obstacle, however, the fact that there are no clear-cut established methods currently in research to identify SCK. Hence, I relied heavily on my own teaching experience and knowledge to identify such knowledge. Additionally, I investigated research

aimed at describing SCK in depth. When an instance of SCK occurred, it was checked with the corresponding definition of SCK and compared with the definition of CCK (Ball, Thames & Phelps, 2005). Most importantly, the classroom use of SCK for each participant was described in full detail below so that readers could inspect the SCK unit of analysis and the context in which it was used. Example SCK units of analysis are shown here below, although simplified for brevity. Full descriptions of actual SCK use are described in detail when the specific case studies are discussed below for each of the participants.

TABLE 23. Sample Descriptions of SCK Use

Teaching Task	SCK use
Responding to students' "why" questions	The postman story illustrates why we might use integers in the real world (the concept of debt, for example). Jake uses this story repeatedly to explain operations on integers in terms of credits or debits.
Finding an example to make a specific mathematical point	Students were struggling with equations of one variable with terms involving x on both sides of the equation. Jake chose several example equations that varied in difficulty to aid students in understanding steps useful in moving forward.
Linking representations to underlying ideas and to other representations	Jake uses the number line, red/blue chips and virtual chips to illustrate the result of computing $6 - -4$
Connecting a topic being taught to topics from prior or future years	Connecting the computation of $6 - -4$ to a problem that continues over the year – the postman story that illustrates concepts behind operations on the integers
Modifying tasks to be either easier or harder	Jake created problems similar to $6 - -4$ but more accessible, building to this example, then moving beyond this example to similar problems Jake knows causes more difficulty (larger numbers, fractions, decimals)
Evaluating the plausibility of students' claims (often quickly)	Responding to students' attempting to disprove $4 - x = 4(-x)$, for all values of x
Giving or evaluating mathematical explanations	Jake asks students if $4 - x$ is the same as $4(-x)$. Jake guides students in explaining that, if $-x = 4$, notice that x and $-x$ are opposites. Equating the expressions on the board, encouraging students to find a value for x and see if the equality holds.

Choosing and developing useable definitions	Jake's example of triangle, square, hexagon building to general definition of a regular polygon (included non-examples)
Using mathematical notation and language and critiquing its use	Encourages and modeled the use of mathematical language such as coefficient, equality terms, expressions and other algebraic terminology when solving equations of one variable
Asking productive mathematical questions	Remember our mail man we used last year and started again this year? Remember what was good or bad when he delivered mail? Subtracting -5 is like taking away a bill, taking away debt, taking away that you owe someone \$5
Selecting representations for particular purposes	Jake uses the number line, red/blue chips and virtual chips to illustrate the result of computing $6 - - 4$

A table was made for each of the 10 participants for analysis of SCK use, although there were cases where a participant's table was either quite lengthy or minimal depending on the amount of SCK use.

Additional Analyses

The remainder of the data involving educational background in mathematics was primarily used to provide a descriptive background of the teacher. At times, this information also played a role in analysis, particularly if the data related to the formation of teachers' beliefs. For example, Julie's educational background was primarily traditional as a K-12 student. Julie relayed that she often fell back on this experiences in making decisions about instructional practices. In Julie's case, her K-12 experience played a role in the formation of her beliefs about teaching and her teaching practices. This was not uncommon amongst participants. However, some participants having experienced primarily traditional mathematics teaching noted holding socio-cultural beliefs about mathematics teaching resulting from a conscious decision to create learning experiences quite different from their own.

Teacher background characteristics were gathered during the focused interview as well. I have listed several questions that were asked in the focused interview:

What experiences from your own K-12 experiences influence your beliefs about teaching and learning the most? What beliefs about teaching influence your practice most? What do you feel helps you feel supported in implementing your beliefs in the classroom? Can you give me an example of how your beliefs influence your practice? Do you hold beliefs that you feel do not influence your practice?

Lastly, the classification of a teacher's mathematical practices was done using the below table again from Raymond's 1992 work. This was done primarily to determine the nature of the consistency of the beliefs-to-practice connection for each participant simply by frequency rates and comparisons with belief findings.

TABLE 24. Criteria for the Categorization of Teachers' Mathematical Teaching Practice

Traditional

- The teacher instructs solely from the textbook.
- The teacher follows lesson plans rigidly.
- The teacher approaches mathematics topics in isolation.
- The teacher approaches mathematics instruction in the same pattern daily.
- The teacher has students engage only in individual paper-and-pencil tasks.
- The teacher creates an environment in which students are passive learners.
- The teacher poses questions in search of specific, predetermined responses.
- The teacher allows no student-to-student interactions.
- The teacher evaluates students solely via exams seeking "right answers."

Primarily Traditional

- The teacher instructs primarily from the textbook with occasional diversions from the text.
- The teacher creates an environment in which students are passive learners, occasionally calling on them to play a more active role.
- The teacher primarily evaluates students through standard quizzes and exams, only occasionally using other means.
- The teacher primarily encourages teacher-directed discourse, only occasionally allowing for student-directed interactions

Mix of Traditional and Socio-constructivist

- The teacher teaches equally from textbook and problem-solving activities.
- The teacher creates a learning environment that at times allows students to be passive learners and at times active explorers
- The teacher evaluates students' learning equally through standard quizzes and exams and alternative means such as observations and writing.
- The teacher encourages teacher-directed and student-directed discourse.

Primarily socio-constructivist

- The teacher primarily engages students in problem-solving tasks.
- The teacher primarily presents an environment in which students are to be active learners, occasionally having them play a more passive role.
- The teacher primarily evaluates students using means beyond a standard exam.
- The teacher encourages mostly student-directed discourse.

Socio-constructivist

- The teacher solely provides problem-solving tasks.
 - The teacher selects tasks based on students' interests and experiences.
 - The teacher selects tasks that stimulate students to make connections.
 - The teacher selects tasks that promote communication about mathematics.
 - The teacher creates an environment that reflects respect for students' ideas and structures the time necessary to grapple with ideas and problems.
 - The teacher poses questions that engage and challenge students' thinking.
 - The teacher has students clarify and justify their ideas orally and in writing.
 - The teacher has students work cooperatively, encouraging communication.
 - The teacher observes and listens to students to assess learning.
-

Results of Analysis

For a student to have exposure to instructional tasks involving specialized content knowledge, teachers must hold sufficient levels of specialized content knowledge as well provide opportunities for students to be exposed to such knowledge within the classroom. Part II of this study looks at two main factors related to SCK use; the level of a teachers' SCK knowledge level and its use in practice as well as how teachers' beliefs related to the nature of mathematics and the teaching and learning of mathematics plays a role in SCK knowledge use in practice. Practically speaking, the following questions apply:

1. How much and of what type of specialized content knowledge do teachers hold?

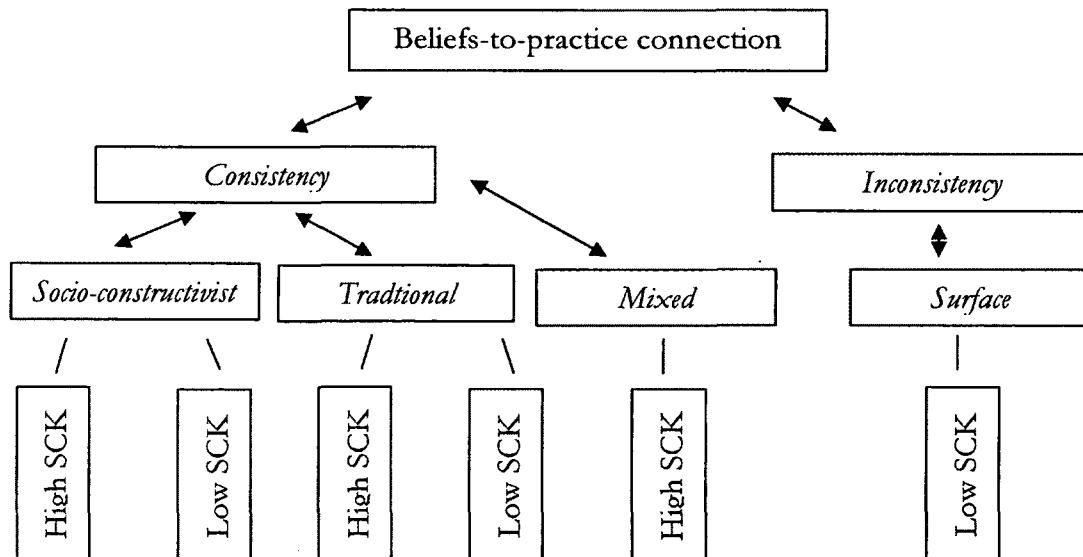
2. How does a mathematics teacher's belief system (specifically beliefs about the teaching of mathematics) ultimately manifest itself in classroom practice?

Two general categories of data emerged as the five data sources were analyzed: SCK knowledge level and the nature of the beliefs-to-practice connection (inconsistent or consistent). There were four main subcategories that resulted from these two more general categories. This section is therefore structured around these four cases, with two major themes as:

- Inconsistent Beliefs-to-Practice Connection: Surface Beliefs (Robert, David & Julie)
- Consistent Beliefs-to-Practice Connection: Traditional Beliefs (Debra & Nancy)
- Consistent Beliefs-to-Practice Connection: Socio-constructivist Beliefs (Sarah & Melissa)
- Consistent Beliefs-to-Practice Connection: Mixed Beliefs (Jake, Anna & Beth) add to diagram

For clarity purposes, I offer the following organizational chart to explain how the themes emerged:

FIGURE 12. Overview of Themes from Analysis



I now turn to the discussion of the first theme, where the beliefs-to-practice connection was primarily inconsistent. The discussion of a participant's beliefs about the nature of mathematics and mathematics learning were included to provide a more cohesive picture of a participant's belief system. At times, information or data for certain participants was either described in more detail than other participants or omitted, for the purposes of illustration.

Inconsistency in the Beliefs-to-Practice Connection: Surface Beliefs

Robert, David and Julie. Initially, Robert, David and Julie indicated beliefs about teaching that were primarily socio-constructivist both within the beliefs survey and focused interview. Their beliefs about the nature of mathematics itself as well as their beliefs about teaching mathematics, on the other hand, were not reflected in their teaching practice. At times, Robert and David also indicated holding contradictory beliefs. For example, on the beliefs survey, Robert indicated strongly that teacher should show the steps of a word problem before students solve problems on their own, yet also indicated strongly that teaching is done best when students are allowed to solve word problems on their own before being shown by the teacher. For Robert, this contradiction remained when probed further during the focused interview.

When Robert and David were pressed for a meaning of their teaching beliefs categorized as socio-constructivist, neither teacher could describe the meaning of their belief; also, it was unclear if either teacher could describe what their beliefs would should like in practice. For example, Robert often referred to how important it was for teachers to focus on teaching *methods and understanding*. When pressed for the meaning of either term, he stated “reform-

based mathematical teaching practices...we need to show the the understanding behind everything.” However, in practice, Robert regularly reminded students to “focus on the method” while he was explaining how a procedure worked; not why the procedure works, the reason the procedure is used or any additional information outside of procedural knowledge.

Robert often used the term “method” when also instructing students step-by-step in how to find an answer to a problem. “This is what you do when you have this situation,” Robert said often during instruction. Robert repeatedly referred to himself as a “constructivist teacher” yet there was no evidence that Robert held an understanding of these terms.

Julie was unable to explain the beliefs she felt she held, with the exception of beliefs about the nature of mathematics, and made few connections between her perceived beliefs and her actual teaching practices. Julie indicated that she did not often think or reflect upon her beliefs about teaching. Julie did, however, emphasize that she held “constructivist beliefs” yet when pressed as to the meaning of her beliefs or how her constructivism philosophy played out in practice, she was unable to provide this information. I now set the stage to describe both Robert and Julie’s practice.

Background and Setting

Robert teaches at a middle school of approximately 1,020 students over a span of grades 6 – 8, in an area that the National Center for Educational Statistics’ classifies as a mid-size city² with a population of 109,565. The student makeup is approximately 85% White, 4% African American, 9% Hispanic, 2% Asian, and 1% American Indian. The median family

² a mid-size territory inside an urbanized area and inside a principal city with a population less than 250,000 and greater than or equal to 100,000 (NCES, 2011).

income of this city is \$48,720, approximately \$2000 below the average for the state of New Hampshire, and the district spends approximately \$9,753 per student, well below the state average of approximately \$12,500. In Robert's school, 31% meet the requirements for free or reduced price lunch (NCES, 2011). There were 19-29 students in Robert's classes depending on the assigned class size and absences during the 3-day observation period.

David and Julie both teach at a small, private school in Massachusetts. David teaches primarily teaches grade six math. David's class has a total of 7 students in the 2011-2012 school year, and all students are male. Julie teaches a primarily grade 7 mathematics course. Julie's class has a total of 13 students in the 2011-2012 school year, with 5 girls and 8 boys. Demographic details of David and Julie's school were unavailable. Other town demographics were not applicable, as many of the students in David and Julie's school do not reside in the local town.

When Robert was asked about the potential influences he perceives from his own K – 12 education, he noted that he graduated along time ago so this information was difficult to recall. Robert did recall and relay that he was not a good math student at that time.

Teaching was the last thing on my mind. I wasn't a good student in math. I was a C student. Maybe a B student in elementary school.

Robert decided to enter teaching mathematics later in life, after 20 years in engineering. Robert holds a B.S. in Technical Business Management and went on to complete 24 hours of graduate course in secondary mathematics education. Two of these courses were classified as mathematics methods courses which Robert describes as focused on pedagogy at the secondary level. Robert is state certified in middle school mathematics. Robert taught middle school for two years prior to his current school, where he has been teaching since 2003. In Robert's first year at his current school, he taught 8th grade Pre-

Algebra and 8th grade Honors Algebra 2. He has since moved to 6th grade teaching multiple levels of 6th grade mathematics, although he noted that he prefers to teach 8th grade.

Robert shared his resume which included various committee involvements such as principal advisory committees, numerous activities involving working with the state department of education on standards alignment, assessment and certification at multiple levels, such as participating in a test item review committees for state and alternative state assessments. Robert is also a member of his district's vertical curriculum committee and various other state and national mathematics teacher organizations, in which he indicates he participates regularly.

Before teaching, Julie worked for about 25 years as an architect following a bachelors degree in architecture design. Julie indicated that she did not have the opportunity to take a math methods course, but took several mathematics courses including Calculus I and II and Differential Equations. In the past, Julie used her skills in architecture to visit local schools demonstrating applied mathematics. Julie has taught at she considers to be "general grade 7 mathematics" for three years.

David holds a Masters of Elementary Education and has worked on educational exhibits extensively for museums throughout the country. David explained that he took one math methods course for elementary school teachers but no college-level mathematics courses. David indicated that he feels strongly about teaching to multiple intelligences and bringing his museum experience into his classroom. David has taught at the fifth and sixth grade level (a combined grade classroom) teaching all subjects, for approximately six years at the same school. David and Julie both indicated that they have had little time over the years to participate in professional development or any continuing education coursework.

Unlike David and Julie, Robert indicated that he felt his school climate did not align with his teaching beliefs.

I don't feel supported here. I think many other schools are teaching for understanding, doing inquiry-based learning but not here. We focus on test scores. I have to cover what I am told to cover but I make sure students are also problem-solving.

Robert expressed frustration with his current school climate throughout his focused interview. In particular, Robert felt that had little time to focus on mathematic teaching related work, but rather spent most of his time in special education meetings or meetings about standardized testing scores.

David and Julie, on the other hand, feel supported in their current school climate by faculty, administration and parents. Julie noted that her school did not provide special education services nor did they participate in the state-mandated standardized testing. Although both indicated a clear understanding of their school's philosophy and goals, both also expressed a lack of clarity in actual school expectations.

Beliefs About the Nature of Mathematics

When I first met Robert, I noticed a very organized room, full of visual resources. His first comment after meeting was that he has not been through a teacher preparation program, but rather learned through life experience. Robert openly acknowledged the pressure he currently feels as a teacher. In my three days with Robert, Robert was called to multiple special education meetings during his preparation time and spent the remaining time trying to catch up on an inbox full of parent e-mails of various content. Robert's openly expressed his frustration with standardized testing, NCLB and other current political

factors. Robert reminded me several times that he would only be teaching for one more year unless he finds a more supportive district. He feels that the climate of the school is not one that supports reform-mathematics teaching.

There is a lot to say for rote. There is also a lot to say about methods, processes, and problem-solving. I don't see that problem-solving is respected around here. We have people who are not math teachers teaching math and they don't have a conceptualization of what math is used for...coming out of engineering, I learned to use math to solve problems. I evaluate problems. Problem solving. Half of my homework is problem solving, not numbers,, numbers, numbers.

Despite this, Robert feels mathematics is vital to a student's K - 12 education.

Math is even more important, in some ways, than reading. Or writing. If you can write numbers, you don't have to write words. I think that math is extremely important. I taught one Algebra 1 lesson in German, showing it a universal language. We talk about where it came from, Greeks, Arabs, Egyptians, Romans, they all had math. Numbers are very important. If it were up to me, I'd stress more math. It crosses social studies, science, all subjects. We read/write across the whole curriculum but we don't do math across the curriculum and it shows in our standardized scores and then we get hit hard for supposedly not teaching math well enough.

Robert views mathematics as relatively static at all levels, but even more so at the K - 12 level. Specifically, Robert perceives the subject of mathematics as a group of methods that must be understood before used.

David and Julie indicated beliefs about mathematics that were primarily instrumentalist. Julie agrees that students learn best from direct teacher instruction and skill mastery.

Students should be able to do the problem. The skill. Most of these kids won't use this after school unless they go into a STEM field.

Beliefs About Mathematics Learning

In Robert's opinion, mathematics learning is best achieved through practice and via this practice, students will develop a toolbox of skills. In addition, Robert believes students should also be able to problem-solve, but teachers should show how simple it is to solve word problems before students solve word problems on their own. Robert indicated that he is unsure whether or not students can solve word problems on their own. In his focused interview, he indicated that students usually have to be shown detailed steps by the teacher to successfully solve word problems.

To be successful in mathematics, Robert, Julie and David indicated that they believed strongly that students must listen well and be shown how easy it is to solve word problems, although all three teachers recognized it was possible that middle school math students can solve simple word problems on their own sometimes. Julie disagreed that students should be allowed to devise solution to simple word problems before the teacher demonstrates the solution.

If I don't do it first, it's a waste of time. They can't usually do it. So if I do it a few times, then they can repeat the steps. Sometimes even that doesn't work.

Beliefs about Mathematics Teaching

Robert agrees that students can find solutions without teacher instruction. David and Julie concur and add that teaching would be best done if students first attempted to find their own solutions on a regular basis. During his interview, Robert added that he believes that sometimes students learn well when allowed to experiment and problem solve, that this was not a good practice in general. He added that he does not have the time to do this with his students due to his current school climate. He stated that he perceives the climate of the school as viewing such activities negatively, wasting valuable time that could be used to prepare for the state assessments.

There is a lot to be said about problem solving and applications of math.

Learning how to apply something. It's about the methods. Not about the numbers. The numbers mean nothing.

It is imperative, according to all three teachers, for students to practice math skills frequently in order to be successful, but should not be a "show-stopper" in student approaching word problems containing the underlying skills to be practiced. Julie explained that she felt students should be given opportunities to construct their own knowledge.

We should give them a chance to find away to represent the knowledge for themselves. They should have chances to solve problems in groups and talk about solutions. I give them real-world problems. Things they can touch and feel. I think all students can learn math if they have this chance.

All three teachers initially indicated mixed beliefs about teaching and learning on the beliefs survey, leaning towards primarily socio-constructivist.

Robert's Use of Specialized Content Knowledge in Practice

Class observations lasted approximately 40 minutes and occurred over a 3-day period. The main content, according to Robert, was adding or subtracting integers, instruction including multiplying integers if time allowed. Despite the complexities of his job, when Robert's students walked in the door, he welcomed them and clearly worked hard to be enthusiastic and engaging. Robert clearly enjoyed each of his moments with his students. Robert began the first class by handing back quizzes. The class was focused on their grade and Robert stressed the importance of their performance on testing. Robert emphasized that answers were right or wrong and feedback was brief.

What did you get? Well, that's not very high for you. What problem are you asking about? Well, that's not -6. It's right or wrong.

Robert then introduced the goal for that day adding integers.

If you know the methods, you can throw the number in and it works.

Multiple ways of doing things, middle school you have choices, elementary school, you didn't. Integer operations – addition, is all about more. For the most part, more is good. Good old zero sits in middle of number line, go more positive, go this way, go more negative, go that way. Going to talk about more and less. Subtract sign, less.

Robert refers to the number line drawn on the board marked with -10, -5, 0, 5, 10 and he suggested to students to visualize the number line if they would prefer.

Add $+3 + +2$, did you have this before? Plain old math, I bet elementary school teachers never told you that both 3 and 2 are positive numbers. This the number line you had in elementary school (Richard points to the positive numbers). Integers really go from negative infinity all the way over to

positive infinity in whole number steps with no fractions, decimals, mixed numbers.

He then expands upon the students' knowledge of positive numbers, from a slightly different perspective.

This, $+3$ is a positive 3 and this is a positive $+2$. Go 3 positive first. Go MORE positive, meaning go 2 more positive. Addition is all about being more. Give me more candy. Give me more jolly ranchers. These are all the neat things you didn't learn in elementary school.

Robert often interjected different representations of the number line, as well as how to represent the integers and operations on integers. The difference scenarios were intertwined within the concepts naturally.

When you have negatives. Positives and negatives are opposites. From 0, go to negative 5. Add negative 3 (go more negative). If plus 3 is more positive, -3 is more negative. If this is 0 and I go 2 positive and I go 2 more positive, I am more positive. Move to the left to add negative. Not too bad if you follow the instructions, start at 0, go to positive 3, from there go more negative by 8. Wind up at -5 . All we are doing is following the instructions.

Robert often used hand signals or visual indicators on the board to add to the understanding of "more" or "less". His lesson was full of analogies that he supported students using in their own words and come up with their own analogies and examples.

Follow the yellow brick road. Follow the instructions. Just like math.

Doesn't matter what the numbers are, just follow the rules in math.

Robert made it clear that zero is a placeholder between the positive and negative numbers and not only the symbol for "nothing". He used zero as a placeholder to

emphasize that a number and its negative are zero pairs. He did not mention the term additive inverse or other mathematical language or mathematical notation typically associated with operations on integers.

Zero pairs, positive 25, -25, adds to 0. A student said “there is nothing there” – 0 is not nothing, it’s something. Go outside on a feb. day when it’s 0 and you tell me 0 is nothing. Add the opposite to zero. We should have had integers earlier in the year, we added the opposite but you didn’t know why. We need this earlier in the curriculum. The sum of opposite numbers is zero. Sum of opposite anything is zero. (ex, 6.2, $5/7$). It becomes zero and goes away.

Robert then encouraged students to add and subtract integers without using the number line and asked students for ideas. He repeated these ideas and expanded upon them to clarify the mathematical comments for the class,

Now uses Mt. Eisenhower to go “up” and “down”. 882 feet +3 feet. How would you talk about adding two things together? Put something in a box to save. Coins to save. Week one you put 2 things in a box, next week you put 2 more things in a box. You’ve saved 4 things, the sum of what you have saved. Adding negatives is kind of hard. I’ll give you an example.

California to death valley, lowest elevation in the U.S. Go even deeper into the gold mine shift. Didn’t you go negative and more negative? We did positive. Class on ground floor, go up one floor, that’s an example. I want to go down town and buy something so I take part of the jar and (came from student) not all of it. Now look at this expression – evaluation or find a solution. Solve for x, all of this is the same. -9 and +3. start at -9 go

positive by 3, now he is walking floor squares in classroom. Wind up at -6.

Finish at -6. That leads us to the set of rules up there. No matter what processes we use, we get to the same place. First rule, add 2 positives, start at first positive, go more positive, end up at sum location.

At the end of each of his three classes, he asked students, encouraging students to respond in unison to aid in memory.

Pairs are what? Positive. No pairs, negative.

Despite the helpful analogies and multiple representations, students were not given the opportunity to engage in a discussion involving this knowledge. The use of actual mathematical language and mathematical representations was limited throughout the three-day observational period. Instruction was generally teacher-centered instruction with a strong emphasis on rules and procedures. Robert's instruction focused on telling students how to go from step to step without engaging students in conversations in the instructional process, with very little use of specialized content knowledge during instruction. Robert's instructional practice was henceforth traditional in nature with limited opportunities for specialized content use.

Julie's Use of Specialized Content Knowledge in Practice

In regards to the teaching and learning of mathematics, Julie believes that teachers should share detailed approach to solving word problems and students should be able to do the math problem in the same manner as the teacher. She completely agrees that students learn math best when following the teacher's instructions. Julie indicated that she believes that students need detailed instruction in order to solve word problems. She believes that basic skills must be acquired first, before understanding the underlying methods

or reasons *why*, with understanding following the practice of skills. She totally disagrees that learning would be best achieved if the students found their own methods to solve problems. Additionally, she totally disagrees that middle school students can find solution to many math problems without help. She also believes that students will become good problem solvers if they can follow the teacher's instructions and be a good listener.

On the other hand, Julie indicated on her beliefs survey that teachers should encourage students solving problems on their own and that she believes somewhat that most students can discover math on their own. As with the other teachers in the study, the beliefs survey is only a starting point when attempting to understand. However, in the case of Julie, she indicated many of the same beliefs in her focused interview.

Julie's observation occurred over a consecutive 3-day period. Like Nancy, Julie also has classes of an hour and 2 minutes in length. Julie was consistent over three days in goal-setting and expectations.

This is a reminder of the area formula. We will do area of a triangle and rectangle although rectangle should be a refresher. Write it down. The more you write down the area, the more you will remember it. Keep writing it so it looks professional, keep writing your units so that at the end you don't forget your units. This is exactly what I wrote on the paper. You can show work on the side. Rectangles first. We'll take figures and break them into different parts and find the area. Well, there are two ways to do this. No matter how you do this, you will always have 2 equations. I will always see two equations on your paper or you will get a 0.

Julie introduced a shape similar to the shape shown below in Figure X.

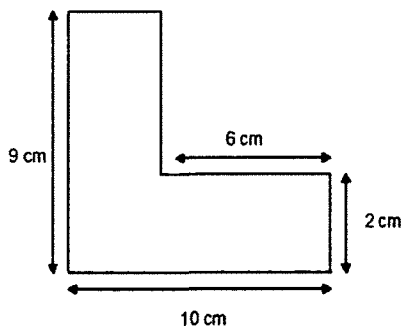


FIGURE 13. Julie's Shape Discussion

Julie told students that there were two ways to do this problem. She told them that they could find the area by taking length times width of the entire figure, with the “missing piece” still there, then subtract the area of the missing piece. The second way to do this problem, according to Julie, was to “split the shape into two shapes, then take the area.”

You can look at it as finding the length and width of the whole rectangle then subtract the other area. Or that's one way of doing it. Now breaking it into two pieces and finding area is another way to do it. Let's listen. I have a whole rectangle and the whole rectangle has an area. I am going to take away the area of the missing piece. Either way, your paper should have two equations on it, written like I did it. On the quiz, if you don't do this, you will not get any credit. You can take that same rectangle and take out this little shape here, I can divide the rectangle in two different ways. So if you do it that way you have to figure out some calculations on your own; figure out that those missing sides are. You will find the area of figure 1 and area of figure 2. You guys try to do that.

Julie observed that some students were not setting up the problem as she did hers. She walked around the room making observations about student work. She repeats what she should see on paper.

I know you just want to know the answer but that's not how it works.

Always write your problem out like this, don't just think 12 times 6 is 72.

Ok, so this is what you need to do (student struggling). Here is the rectangle.

Here is Area 1. Here is Area 2. Now how do we find Area 1. Now, find area

2. Now add area 1 and area 2. Where are your units. It's going to be a pass

or fail grade, you know how to do it this way (the written out, method) or

your fail. You think it's simple, If you want a 100 or 0, you decide, I am

giving you 2 minutes to copy. Just pick a section and start.

Julie reiterates the formula for the area of a rectangle and writes it on the board. She reminds students of the goal and procedure.

I want to see $A = l \text{ times } w$, plug in, then with feet squared, this is what an algebraic problem should look like and nothing other than that. Everyone has to have this down, area of a rectangle. That's how you do it, that's what it should look like. There are lines on your paper for a reason. You should then have straight lines for your rectangles. From now on if I don't see this, you will lose points. I am pushing for more goals and challenges for you.

Repeats entire procedure. Units squared if it's on the graph paper or if they don't use units. Make your lines as parallel as you can. You just fill in and substitute, you can't change any format.

Julie again walks around to check on student work and stops at one particular student to discuss his work.

Ok what did you get for the first areas. For area 2, what did you get for that. So it's the 3 times the 10 so in the first one, you are finding this rectangle, then you find this one. You did this one right (speaks to an individual student) but what you want to do with this one is break it into two rectangles, not 3. You don't want that. I mean it's ok to do that but you really don't want to.

Julie offers the class a similar example but with different numbers. A student asks quickly what the answer to the problem is.

I don't know what the answer is but you have to do it in the right format or it's not right. For each problem, you should have the same thing, just different numbers, same format. Again, you should have two sets of equations. No matter which of the way you do it. If you don't have that, you haven't learned anything today. Does everyone agree with this? Here we go, put this example, 24 feet by 13 feet. I'll go easy on you and make this 10. This problem should take you about 15 seconds and if you don't write it how I just showed you, you will have to do it over again. Goes around the room helping, telling steps. If you multiple any number by 10, you should already know without doing the work that the number gets a zero added to it. Now, same rectangle.

Julie moves on to talk about the area of a triangle but first begins by reviewing basic concepts related to working with triangles.

Now let's find the area of triangle. All triangles are treated the same way, basically. The height and a base, base what it sits on, height has to be perpendicular. How many degrees in a perpendicular angle? I need to drop

a line down to the base sometimes. If you have an obtuse triangle, the height is here, perpendicular to the base. Day 2. If your length is 12 feet, width 13, feet, write your problem just like this. I want to see $A = l * w$, plug in, then with feet squared, this is what an algebraic problem should look like and nothing other than that.

The last observation day started with the class reviewing homework from the previous evening which was an assignment to correct the last test's corrections. One question at a time, she asks if anyone prefers to go over the problem.

This is a function box – put the x in and y comes out. Always set up this way, x on left, y on right. I guess we can add types like x bananas. This is a basic function box. If you put in one for x , you come out with a 5, 2, 10 so if you set up a formula with x relating to y , If I am adding 5 each time that means I have $x + 5 = y$. So if $x = 1$ then $y = 6$ and it's not. So it's times 5. For every number I substitute for x , it's that number times 5 is what works. If you were to make an x - y equation for this it would be $x * 5 = y$.

Julie continues in asking if students have further questions on the homework.

Do you want to go over a reducing one? If you have a problem like this, 480 over 360 and the problem says to reduce? What is the first thing you should be doing. By crossing off the zero by dividing by 10, by doing this you are lopping off the zero. 10/10. You could do 4 over 4 which will get you 1 over 9 and that would be your answer. Get the largest number when reducing that goes into the numerator and denominator. Lop off those zeros right away otherwise it will take you so much longer. Say you have 400 over 540, lop off two zeros.

Student is trying to reduce by 6, she says no, it doesn't go in, says we need to reduce by 2.

She then moves onto finding an angle "inside the circle."

Here's the center of the circle. Someone tell me the radius. What is the radius. Encourages student to come up and show the radius. Who can describe that differently. Another student tries, Third student tries,, then she says, now that's correct. It's not a corner of a circle jus a line that goes from the center to the outside, the diameter cuts all the way through. If you have these cords that come from one end of the circle to another, it forms angles, called inner angles, central angles. If I have cords in such a way that forms angles formed on the outside are inscribed angles. So on that problem, you need to find the cord is a diameter, you want it to have this point, this point and this point and in that order. BC or CB. With the angle, you need to have 3. A central angle, find that, which one is acute? That's what they are asking. So you always want to put the letter of the angle in the middle, put the little angle sign in the front. What else could I call it? What about 14? Even if you got the problem right, you need to follow it because you want to keep going over them to keep getting it right because you will see them again and again.

During the last few minutes of the third day of observations, the class ends with Julie discussing one last homework problem.

What kinds of numbers are these? Mixed numbers. Do we need common denominators? No, we don't need them. We need improper fractions.

Julie again walks around to check student work. One student is still working on an earlier problem involving area similar to Figure X.

Something isn't right here, how Evan did this, does anyone see it? Show your method up there. What should be the next thing, this is not complete. Need the second equation. Other way to do it is what Kevin did. You still need to have two sets of equations.

Julie encourages to use the words height of a triangle as "perpendicular distance." In responding to a student reaction, she makes the following comment.

I am not accusing, I am helping you learn to speak math terms. The height of the triangle is the length that is perpendicular to the base.

Julie primarily focused on teaching by telling, an instructional practice typically associated with traditional mathematics teaching practices. Hence, there were minimal opportunities for students to engage in the use of actual mathematical language. The use of mathematical representations was limited throughout the three-day observational period. Instruction was generally teacher-centered instruction, and focused on telling students how to go from step to step without engaging students in conversations in the instructional process, with very little use of specialized content knowledge during instruction. Julie's instructional practice was henceforth traditional in nature with limited opportunities for specialized content use.

Summary

Both Robert and David indicated that their expressed socio-constructivist beliefs about teaching and learning were beliefs that they *should* hold, yet in the classroom, Robert, David and Julie enacted traditional mathematics teaching beliefs. All three eachers revealed in the focused interview that their tendency was to fall back onto their own K-12 traditional teaching experiences, particularly when the the pressures of time, standardized testing, student behavior and the involvement in the process of special education services as

constraints. They additionally indicated also that ultimately they felt traditional teaching methods were best for their students because this approach worked for them in their own earlier experiences as students.

There are often significant mediating internal and external factors, such as school climate, influencing mathematics instructional practice, particularly influencing the link between beliefs and practice (Pajares, 1992). Specifically, the role of the external factor of a teacher's school climate on constraining or enabling teachers' beliefs is well documented in research (Stipek, et al., 2001; Handal, 2003). It is possible that when such constraints arose, these three teachers fell back on their traditional teaching practices, indicating that their traditional teaching beliefs as more central than their surface, or less central, non-traditional teaching practices (Green, 1971).

All three teachers were similar in that they appeared to hold what would be considered surface beliefs. The three teachers all indicated that they generally did not have time to reflect on their teaching practices or their beliefs. For Robert and David, beliefs of a socio-constructivist nature were what these teachers felt they *should* hold. It is interesting to note that these were the only three participants who indicated that their teacher education program did not specifically include a course or courses aimed at developing teachers' philosophies about teaching and learning. All other participants in the study pointed to different respective courses during their teacher education program in which they focused on developing their beliefs as teachers.

Lastly, all three teachers had an inconsistent beliefs-to-practice connection, with the exception of their beliefs about the nature of mathematics, primarily using traditional instructional practices. In addition, all three teachers scored on the lower side of a low-average range on the MKT survey, yet it was unclear during teacher observations if the

teachers instructional quality was a function of a lack of knowledge, instructional practices or both. Ultimately, Robert, David and Julie provided students with little or no access to specialized content knowledge, quite clearly limiting the quality of mathematics instruction during the observational periods.

Consistency in the Beliefs-to-Practice Connection: Limited SCK Use

Debra and Nancy. Both Nancy and Debra's beliefs about mathematics and the teaching and learning of mathematics were categorized as primarily traditional. Both teachers referred to their school climate and other contextual factors as supportive, particularly with respect to enacting their beliefs within the classroom. Debra and Nancy both had a generally consistent beliefs-to-practice connection with primarily traditional instructional practices over the 3-day observational period.

However, Debra and Nancy differed significantly in their level of specialized content knowledge. Nancy demonstrated low specialized content knowledge mathematics for teaching via both the MKT survey and structured interview SCK tasks. Debra, on the other hand, demonstrated high levels of MKT on the MKT survey and the structured interview SCK tasks. Despite this significant difference, both teachers provided students with little or no access to specialized content knowledge, clearly limiting the quality of mathematics instruction during the observational period. When probed further, both teachers pointed to their traditional teaching beliefs as the most significant factor in their choice of classroom practices.

This understanding is particularly significant. Proxy measures are often used as indicators of teacher quality, such as teachers' subject matter knowledge and the number of

mathematics courses taken, for example (Hill & Ball, 2009). The case of Debra and Nancy, however, points to other factors in understanding teacher quality. Debra has completed a similar sequence of coursework as a mathematics major and clearly demonstrated knowledge of mathematics, yet provided students with little access to specialized content knowledge; leaving use with limited information to judge teacher quality. Clearly, a high level of mathematics knowledge, even the unique case of a high level of specialized content knowledge, is not necessarily sufficient for quality mathematics teaching. Teacher beliefs played a substantial role in shaping the teaching practices of Debra and Nancy, limiting the opportunity for specialized content use, suggesting that the assessment of teachers' beliefs warrant consideration in the determination of overall teacher quality.

Background and Setting

Debra primarily teaches algebra for grade 8 students at a public middle school in New Hampshire encompassing grades 7 and 8, in an area classified as rural/distant, i.e. rural territory that is approximately 5 to 25 miles from an urbanized area. The entire grade 8 has approximately 91 students and Debra typically has a class size of approximately 20-22 students. The median family income in the area is \$87,680, significantly higher than the state average. The current per pupil expenditure is slightly above the state average at approximately \$13,000 per pupil. Debra's school is primarily Caucasian (NCES, 2011).

In addition to teaching grade 8 algebra, Debra also teaches one section of a high school algebra class in the same school district's high school. Debra is an experienced middle and high school teacher, having taught middle school for 7 years and high school for 15 years, including courses such as Algebra 1, Algebra 2, Geometry, Pre-calculus, Trigonometry and Calculus.

Nancy teaches one grade below Debra in grade 7, at a relatively new pre-K-12 private school in Massachusetts. Nancy taught adult education math courses on and off over 10 years and has no high school teaching experience. This is her first year in teaching middle school mathematics in a grade 6 – 8 middle school building with a much smaller pre-K – 12 school population than Debra’s public school; Nancy’s middle school has 23 students. Like Debra’s school, the population is primarily Caucasian. The per capita income per family information, student expenditures and other particulars are not available. Nancy’s class and course load is remarkably smaller than Debra’s; Nancy has a total of 11 students in the 2011-2012 school year, with 4 girls and 7 boys.

Debra and Nancy indicated similar views regarding their teacher education experiences; with the exception of their descriptions of what they perceive were math methods courses. Debra completed her masters approximately 20 years ago, through a school she refers to as a “teaching school.” Debra explained that all of her classes within her master’s program, including both math, math methods and education courses, were not of help to her. She referenced 9 or 10 mathematics courses which she indicated as being similar to those taken by mathematics majors at that time. She recalls that she took 2 courses she would classify as math methods, and had no formal teaching internship. Debra acknowledged minimal usefulness of her methods courses, but pointed primarily to teaching experience as the most helpful in her teaching career.

Nancy received her masters through an on-line education program in mathematics education in secondary education. Nancy took Finite Mathematics, College Algebra, Pre-Calculus, College Geometry, Calculus, and Secondary Mathematics. Nancy took one methods course and unlike Debra, she described this class as the most helpful course she took because, she explained, it focused on actual teaching practices.

If I didn't have that one methods course, the MKT survey would be really hard for me. I only had one methods course in my masters program. It was all math. I wish I had more than one methods course, that was my most helpful one.

Nancy's degree program ended in a research project and like Debra, Nancy indicated that she has no formal teacher internship.

Both teachers indicated feeling supported within their respective school climates and by the surrounding community. Debra relayed that she feels this support is conditional, relying heavily on achievement testing results. Despite indicating potentially conditional support, Debra felt strongly that this did not influence her teaching practice.

It's a good district for math. Parents are supportive. I am supported because my kids do well on competencies and standardized tests. They leave me alone. I don't know what they would do if the kids didn't do well. I am not worried about it...I teach this way because it's what works.

Nancy described her school climate as one that is highly supportive and allows for creativity in teaching. Nancy feels there is very little structure and no curriculum guidance.

The school supports whatever I want to do but no one is ever in my classroom, except for when there are student tours. I have had to come out of the current curriculum to teach functions because the book doesn't develop concepts well. I have been told to stick to the book which is good but I have to come out for functions. I don't know...I guess the curriculum works for kids who are not ready for Algebra 1. Very limited resources here, especially with technology. Not always a lot of communication with

administration though. I guess I am not clear as to what the expectations for teaching are here but I do feel like I am supported.

Nancy believes that her earlier life experiences have influenced her beliefs about teaching and learning mathematics in some ways. Nancy describes her high school mathematics experiences as traditional and difficult to understand.

I never understood high school math until I took physics with a teacher that made connections and showed me it was actually useful. I can parrot things back but wanted connections. I taught because I thought I could do it better. When I was about 30, I got my teacher certification for math, grades 7-12, in Michigan. I taught adult education, alternative teens in the morning. Basic math, a bit algebra and word perfect.

Despite her perceptions of her own traditional experiences as negative, Nancy's own beliefs system, described next, reflected similar traditional beliefs. She goes on to describe how she came to teaching.

Several women friends of mine in the same area were told to drop out several times. I was a kid so I didn't know what to think. I had to pick a major and graduate. But first I took 2 years off and became a ski bum. I really didn't want to do engineering. Mom was a teacher of elementary, use to go into her class and go into her class on break. There were never any teaching jobs but I did want to teach at this point.

Debra has little recall of her own K-12 experiences; with what recall she had, she indicated that her experiences were primarily traditional and typical of how students should learn mathematics.

We had the textbook...we listened to the teacher, went home and did the problems, came back and did it again. That's how we learned...we listened and practiced.

Debra's beliefs about her own educational experience resonate with a regular them in research on teacher beliefs that teachers acquire many of their own beliefs from their hours and years of observations (Carroll, 1995; Thompson, 1984).

When Debra was asked about how she came to teach mathematics, she stated that she was "going to be blunt."

I chose teaching because of the schedule. I was a business major and I did a lot of helping other students. I would not have picked teaching... I was one of the smart kids. Would have been embarrassing to say I was going to be a teacher. I found teaching fun, though.

Beliefs About the Nature of Mathematics

Both Debra and Nancy indicated beliefs primarily representative of an instrumentalist view of the nature of the mathematics discipline in both the beliefs survey and focused interview. Debra states:

Math is the same everywhere you go. Math is math. It's a set of rules that are math. Math is just math. It doesn't change.

Debra relayed that that mathematics was "mostly facts and rules." Debra consistently indicated that mathematics is a subject that does not fixed and absolute, stating that "there is usually one answer." During her structured interview, when asked how she would know a student's answer to a problem was correct, she stated, "well, it's either right or it's wrong. I would check to see if it's right or wrong."

Nancy indicated several strong beliefs regarding the nature of both the beliefs survey and during her second interview.

Math as a subject... there is some change like over hundreds of years, but more so changes in mathematics education At this level, K – 12...there is not a lot of change in the curriculum, but changes as to how we teach it. I try to make it relevant but it's really hard. It's pretty black and white.

Nancy explained that she determined whether a problem was correct by determining if it was right or wrong.

They can try all they want but if it's a standardized test, it's going to be right or wrong. I'm not teaching them if I am teaching them they can make the mistakes and not lose the credit.

Beliefs About Mathematics Learning

Although Debra indicated that she valued group work, she primarily described activities in her classroom in which students worked individually. She often referred to the importance of practice for a student to learn a particular skill. Debra relayed that some of her students were able to handle an inquiry-based classroom activity and but most cannot. When asked if whether all students can learn mathematics, Debra stated the following.

Some kids are really just born with it. Some kids can look all day, you can explain all day and they won't get it. I don't think it's like if a parent isn't good at it, then the kid isn't good at it. Effort is almost equally as important.

When you go to math competitions, it's never the kids that need that effort that do well though. Some kids just have it and some don't.

Debra further explained that for students to learn mathematics well, they must be given explicit instruction “because most students cannot figure it out themselves.” When pressed during the focused interview about what she mean by student understanding, she relayed, “students should know how the algorithm works.”

Nancy’s belief system was similar to Debra’a. Nancy explained that ultimately, most students need explicit instruction to learn mathematics well. Nancy also agreed that students learn math best from the teacher’s presentation and application. Nancy explained that students learn best when listening well to the teacher and “following the steps.” She relayed that she felt the practice of skills and mastering the steps of algorithms as primary indicators in the success of student learning in her classroom.

Beliefs About Mathematics Teaching

Debra explained that teaching should include being explicit about each step and pointed to the textbook in aiding this endeavor.

Textbooks give examples. We give examples. It helps them to see the steps and different examples, then practice the problems at the end. That’s how I learned math and it works. Textbooks haven’t changed that much.

Debra explained how she generally stuctured her classs, starting each day with the problem of the day, followed by instruction and possibly a quiz. Debra pointed to providing students frequent opportunities to practice and a consistent daily routine as key teaching practices. Specifically, Debra explained that students should see the same problem repeatedly in order to understand the problem.

They need to see detailed steps. Most of them can’t figure things out – some of them can.

Nancy also feels strongly that practice of skills should be a central piece of her teaching and demonstrated by her. She explained that it is helpful to explain to the algorithm to students before tackling a problem to solve on their own. When asked if the understanding of a concept is important, Nancy stated:

I wish I had more time to ensure that students understood a concept. But there's never enough time, to get out the manipulatives and fool around with them. Before I know it, the class period is over and then they've learned nothing. At least if they are able to do the steps of the problem...do the problem...they can pass the test.

Debra's Use of Specialized Content Knowledge in Practice

Due to Debra's scheduling preferences, observations were completed over a full day instead of three separate days. Debra had 3 sections of the same level algebra class during this day, as well as two sections of a pre-algebra class. Debra structured each class similarly. Debra was preparing students for a test, as well as exposing students to an additional new concept. At the start of each class, students watched an overhead projector while Debra presented a table in which students calculated their current grade based upon two newly returned homework assignment grades. At the end of each class, Debra gave students a practice test and solutions for exam preparation.

You are going to get two assignments back today that we will record on the gradesheet. First record binder quiz. You have one grade quiz here so far. It has a weight of one, put it in the second column. Record your slope and graphing classwork grade. So, this should be a 1 and that should be a 1, the weight of each assignment is 1. So if you remember the steps. Let's get your

grade recalculated. I am going to pretend I am a student and put in my own grades and calculate this.

Debra then begins going over homework problems from the night before. Students have a polling system which indicates student responses. She polled to see who got each question correct. If less than 70% of students indicated an incorrect answer, she choose to review the problem step-by-step on the overhead screen. Debra reviewed a problem involving the slope-intercept equation for a line.

You want it in the form $y = mx + b$, you want the term that has the x written first. Here is your slope and here is your y -intercept. First thing you want to do is write this so that you have the x term first, but the $-6x$ has a negative 6 and $-6x+4$ means a positive four for your y -intercept. So be more careful next time (students note errors) I didn't know you could switch them and keep the sign. $4-6x$ can switch to $-6x+4$ (student).

Debra relayed to students for them to stop her if they had questions. She instructed them to start a new page in their binder and add today's date and take notes while she talked. She moved on to the next problem.

First problem, is the point a solution to this equation. If you don't remember, look at your notes. This is going to be the first problem on your test. If this is a solution to this equation you should be able to put the 2 where the x is and 3 where the y is and get the right answer. So this point is sitting on the line somewhere. That's what this means. So both sides of the equation are the same, it is a solution.

She then asks students to use the slope formula to calculate the slope of two points on the coordinate plane.

So slope, here is the formula and I will put the formula on the board. This is the formula. Yes, you think you got it right, poll yes. If you got it wrong, poll 2. She goes around to look at student to check answers. First thing that you are doing wrong is that you are not labelling your ordered pairs correctly. Here's how you label them, x_1, y_1 , then x_2, y_2 .

Debra is checking student work and states that she notices some students are putting the coordinate into the formula in a different way than she proposed.

Should they come out differently if they are put in a different order? If you switch one and not the other, it's like a different ordered pair. If you switch them both...She quickly calculates with the coordinates switched, then negative sign just moves to the top in this case. Some of you lost your subtraction sign.

Debra then asks students to find the slope of that line and y-intercept, then graph the equation using the slope and y-intercept. Students are working individually, Debra is checking work.

Tell me where the slope is. Tell me where the intercept is. Where do you start. Slope is sitting next to the left of the x. Always start at your intercept. Do rise over run to create your line. Put the equation in slope-intercept form. Get the y by itself. You can't just move terms, you have to subtract the $2x$ which is how you get rid of the plus $2x$, remember everything you do to one side is what you do to the other side. Don't put the -5 first, put the x-term first. If it's a plus here, yes, you will always subtract. If it's a minus, you will always add, yes. When you find the slope, should you have the x there when you talk about the slope? No, just the coefficient of x, that is the slope.

She then gives students an equation that she asks them to put in slope-intercept form.

Put it in y-intercept form, then tell me the slope and y-intercept. Goes through procedure for students, step by step. If you leave out this x you will have a horizontal line. You can't just leave out the x because it's very important. Be careful with your signs. Here's the problem, sign mistakes all over the place. This 4 is a negative 4. If you lost that sign, your signs are messed up and you have the wrong answer. The equation should be $y = \frac{3}{4}x - 11$.

Debra asks students if there are solutions to the following equation and if so, how many solutions does the equation $y = 3x + 2$ have?

Lots. Infinitely many solutions, the amount of solutions is infinity – but infinity is not a number. Make sure that you always move the x term to the right hand side and y to the left. Put your y and x into the equation to see if it is a solution. We've been graphing lines, right? So $y = mx + b$ is a line. What does the point mean, the solution. So when you graph your line, when you extend it, it goes through this point. The point is sitting on that line.

Debra responds to a student question about what slope means when the denominator is zero.

Here's a problem, we already know a vertical line has an undefined slope. But when you compute the slope, you get 8 over 0. Look above. The operation doesn't make any sense in math. [Insert Debra's proof].

Debra hears a student tell her that she has the square root of a negative value as a solution to a problem.

Student question (if -4 and -4 is a square root of 16 , she says but what is a number that we can square to get -16). Cannot change a rational number is an irrational number unless you are multiplying it by an irrational number.

No imaginary numbers are square roots of negative numbers.

Debra used specialized content knowledge occasionally when students asked for an explanation. In one example, she used a proof in her explanation. Debra's primarily traditional beliefs and teaching practices seemed to be consistent, primarily focused on teaching by telling, an instructional practice typically associated with traditional mathematics teaching practices. There were few opportunities for students to engage in the use of actual mathematical language, and when these opportunities occurred, student responses were not used or built upon. Instruction was generally teacher-centered instruction, and focused on telling students how to go from step to step. However, Debra was clearly able to use specialized content knowledge when the infrequent opportunity arose. Debra's instructional practice was henceforth primarily traditional in nature with few opportunities for specialized content use.

Nancy's Use of Specialized Content Knowledge in Practice

Nancy's classes meet 4 days a week for 1 hour and 20 minutes, and one additional day per week for an hour, with the latter aimed at a weekly set test period. On the first day of observations, the class began with approximately 15 minutes of a discussion regarding a test. Students were given solutions to reference on paper. Nancy responds to a student question about how to do a test problem.

Ok what we do for your tests. You redo the problem. The best thing you can do is learn from it. Look at the solutions, mark which ones are right, the

right a reflection about what is wrong. I will count this as a homework assignment.

There is an ongoing discussion of how students did in the test, discussing points, whether the question was right or wrong and how many points were given for each part of the question. In responding to a student who had a conceptual question on the test,

Some of you did really well. If you have questions, snack-time is a better time to grab me and ask me for help. Now isn't the time, we have to get to instruction. I give you a pattern, so the quiz tomorrow is going to be just like the homework, I give you a pattern. I am going to ask you how many are in step 10 and explain how you find that. You can't just draw an arrow to number 10, you will have to show some work like I did last week and some today. We'll practice this some today.

Nancy then tells students that today, they will be looking at patterns, functions and graph them.

Remember the first function A ...if not, just look up here. There were squares...1, 2, 3 adding another row on step there, then dots up there? We looked at the step number and blocks and put them into a t-table, with a step number we call n . The number of blocks will be f of n . Have you seen this notation? No, well it's new to you.

She then tells students to note that the pattern is an increase of 3.

Increasing in a pattern of *plus 3*. How can you figure out a formula for step n ? (Nods and "uh-huh" at student mathematical productions). Pattern is $3n + 1$. Look at what is changing, that is what we multiply our step number by,

but step one is different. Is the plus one what we are starting with? There is a nicer way of thinking about it.

Nancy then moves on to demonstrating to students how the function can be graphed on the Cartesian coordinate system.

Am going to give you functions and graph paper. This one is easy. This is just like coordinates, the first coordinate where everything is easy and positive. Does everyone know x or y -axis? Make sure your zero is down here and the first one is one. We're going to take this t-table and plot it. The n is down here, and the f of n is down here. I am going up by 1, 2, 3, 4, 5 (counting). 5 is enough. Good observation. My n is over here and my f of n is over here. I'm going over 1, up 4. Does anyone see if you can get to this point from that other point? Good.

When Nancy hears students offering other possible ways of approaching the problem, she refocuses the class on following her own procedure.

When we are doing these points, just do over 1, up 3, otherwise it will be confusing. (Responding to a student who wants to start at zero). These functions make a line but what happens at zero. We can extend the line to zero, goes through zero so we want to make it go through there. This is called a step zero, so I am taking away three each time. At 0, it is 1. The picture matches the function and the function matches the t-table.

Nancy then guides the students through another function w and the steps she uses to find the pattern that will lead students to writing an expression for the function.

Look at function w . Look at the board, w looks like this....step 1, step 2, step 3, look like this. The first part is to make the t-table, when we use f of n

notation where n is the step number and we considering the number pieces.

How can we find a formula for this? What is going on here? You can see it's increasing up here on the board. On step 10 it will be 2×10 . If we look at step 10, what is n ? If we have 2 groups of 2, 3 groups of 2, and so forth so is it $2n$ or $2n + 1$, and how do you get the $+1$? We are looking at groups of 2 because we are increasing in groups of 2 each time. That was why, let's look at x . x looks like this, here is step 1...here is step 2 and here is step 3. On step 1 we have 4 pieces and so forth. What will happen for n ? You can see from the t-table that it is increasing each time, groups of 3. So what's the formula? You can also see this on the graph.

Nancy offers another example of a function y .

Now function y . Looks like groups of 5. More like pentagons, 5 pieces but I can't really draw it. If we look at our n , f of n , step 1 has 5, step 2 has 9, so this is increasing by 4, right? And you can see it in the picture, see step 1, increasing by these 4 each time, so we can use a short-cut way to find the formula for step n to be $4n$ plus 1 because you have a four there, plus 1. Now we'll take a short quiz. Put your stuff away. I am looking for you to write out the math. What you added or multiplied together. I want to see reasoning with it, so show me you added or multiplying how you came up with your formula. You need a t-table or a graph.

After returning a recent homework assignment on Day 2 of observations, a discussion ensues for about 20 minutes as to what problems are correct and which problems are wrong.

What is your formula if you have it wrong? A complex formula is ok, but you need to have it in the simplest form to know what the slope and

intercept are from the formula. What you would have to do, you would have had to get the slope and intercept from the graph. I don't expect you to do this, but you would have to use some algebra here. We would use the distributive law. The times three doesn't work (responding to a student). So then you need to combine like terms. This is an example where the distributive law is useful. We've been using it in isolation. Now you can graph this so use some points. (names some points) make a nice line with the ruler. The y-intersection is your 1. That was all you had to do, quick assignment. Second problem. Same thing. What's your formula (she didn't have it written down).

When Nancy offered a problem to the class and received no response, she would work the problem from beginning to end on the board. She now switches to a related topic of functions.

We've been looking at patterns and doing a formula. Now I give you a formula, and then do a pattern. Need to do three steps for this. So the idea is to have it in some type of pattern. Take out paper and do this one.

Student comes up to board after about 10 minutes of individual work (with no help). She guides...so what is our step 1....ok that's it, then step 2 is 4.

No, don't go up, we only had to do 3 steps. So look up here. There are a lot of examples, as long as you meet the number requirements, you are right. As long as you have the correct number it is correct.

When a student constructed what Nancy felt was an atypical pattern she notes that the student is incorrect during the first step of constructing his pattern.

It should be growing like in a predictable way. See what you did over here, the number is right and nice try, but you added too many here in step 1. You got tricky, five hearts as one. This is an example when you are adding, the numbers match, but not a predictable pattern, but I guess it is because ...in the degrees of things, you could kind of count it in a pattern, but not the best one. Oh wait maybe this is correct but you did it in a tricky way. This is what the quiz is on so you need to be able to do it right.

Nancy's instructional practices primarily focused on teaching by telling, an instructional practice typically associated with traditional mathematics teaching practices. Similar to Debra's classroom, there were minimal opportunities for students to engage in the use of actual mathematical language. The use of mathematical representations was limited throughout the three-day observational period. Instruction was generally teacher-centered instruction, and focused on telling students how to go from step to step without engaging students in conversations in the instructional processs, with very little use of specialized content knowledge during instruction. Nancy's instructional practice was henceforth traditional in nature with limited opportunities for specialized content use.

Summary

Debra and Nancy clearly held similar beliefs about mathematics and the teaching and learning of mathematics that consistently acted as a "filter in their decision making process" as Clark & Peterson describe in their 1986 work involving teachers' beliefs and their influence on teachers' thinking about their practices. Both teachers held primarily traditional teaching beliefs and consistently chose traditional teaching instructional practices. However, the specialized content knowledge held by each teacher was significantly different. Debra

held high levels of specialized content knowledge and when pressed by a student for such knowledge, occasionally used this knowledge in practice. Nancy displayed little evidence of specialized content knowledge over the five data sources, including classroom observations, the ultimate forum for student exposure to such knowledge.

This particular case suggests a consistent, primarily traditional beliefs-to-practice connection may limit the use of specialized content knowledge in practice, regardless of the level of SCK. Both Debra and Nancy were highly teacher-centered, typical of a traditional teaching lens, allowing little opportunity for student exposure to SCK, limiting the quality of mathematics instruction. Beyond the mathematics knowledge held by teachers as a quality indicator, these understandings suggest that the role of teacher beliefs warrants further exploration in future consideration in understanding teacher quality.

Consistency in the Beliefs-to-Practice Connection: Opportunities for SCK Use

Sarah and Melissa. Sarah and Melissa's beliefs were primarily categorized as socio-constructivist; both teachers demonstrated significant consistency in the articulation and enactment of their beliefs typical of a socio-constructivist teaching lens. Sarah and Melissa both indicated consistently that they valued instructional practices typically associated with a socio-constructivist view of teaching and learning mathematics and this was also quite evident in their teaching practices over the three-day observational period. However, both Sarah and Melissa demonstrated low levels of MKT via the MKT survey, although both revealed some use of specialized content knowledge in the structured task interview, albeit limited. When taking a closer look at the MKT survey results, both teachers scored *substantially higher on core content knowledge items than specialized content knowledge items.* Specialized

content knowledge items have consistently been found to be more difficult than core content knowledge items in multiple studies, one of which included a nationally representative U.S. sample of middle school mathematics teachers (Hill, 2007). Not surprisingly, the use of specialized content knowledge was rare during the observational period and hence limited the overall quality of instruction. These teachers, however, seem to have an important skill; bringing about opportunities for specialized content knowledge use.

Background and Setting

Melissa teaches pre-algebra in grade 7 for a public middle school in New Hampshire including grades 6-8 and a total school population of approximately 1200 students, each grade having approximately the same number of students. Melissa's school is located in an area classified as rural/fringe, with a per-pupil expenditure of \$9,700, below the state average. The median household income in Melissa's school district is \$76,980, well above the state average. Approximately 100 out of 1234 students qualify for free and reduced price lunch Melissa's school and the school population is primarily Caucasian (NCES, 2011). Melissa taught grade 6 for two years and title-one mathematics at different schools previously. She is in her first year of teaching 7th grade at her current school.

Sarah has been teaching 7th grade longer than Melissa, for the past 7 years. Sarah also teaches at a public middle school in New Hampshire comprised of approximately half the number of students at Melissa's school, in a rapidly growing town also classified as rural/fringe (NCES, 2011). The district spends \$14,608 per student, above the state average, whereas the median family income is slightly above the state average, at approximately \$61 000. Of the middle school student population, 10% meet the requirements for free or

reduced price lunch. As with Melissa's school, the school population is also primarily Caucasian (NCES, 2011).

Melissa received her Master's in Elementary Education approximately 6 years ago. She noted that she took one methods course in elementary mathematics teaching and was required to take a one-semester internship, both of which she felt were extremely helpful but did not sufficiently prepare her for her first year of teaching. As a result, she went back to take five graduate-level methods courses within the topics of geometry and measurement, numbers and operations, data, probability & statistics, teaching everyone algebra, and an integrated math/science methods course. She relayed that she felt these courses were extremely helpful in her teaching.

Sarah, on the other hand, graduated with a bachelors degree in mathematics for middle school mathematics teaching, a degree program with a heavy component of mathematics coursework: Calculus I , II, Discrete Mathematics, Introductory Statistics, Data Analysis for Teachers, Geometry for Teachers, Programming, Linear Algebra/Differential Equations (combined topic course), Algebraic Concepts for Teachers, Number Theory, Technology for Teachers, and Mathematical Thinking. Sarah had one mathematics methods course which included teachers from all K – 12 levels. This was an opportunity, according to Sarah, to connect with teachers at other levels to talk about teaching math and evaluate multiple national middle school mathematics curriculums such as Connected Mathematics. She finished her degree plan with an internship that lasted approximately one semester, although it began in early January and ended in June.

I had an amazing cooperating teacher. That's the difference no matter what teacher education program you have, no matter how long the internship is.

If it wasn't for her, I would not be the teacher I am. We talked about everything. I was totally ready to teach.

Although Melissa strongly indicated beliefs of a socio-constructivist nature, Melissa felt that she was not fully supported in enacting her beliefs. However, she explained that she takes daily risks to teach as she feels is best for student learning. For Melissa, this meant enacting her socio-constructivist belief system.

Kids have a thirst for knowledge and if you let them play around with numbers, they will make their own connections. Unfortunately because of the current pressure of new standards, there isn't enough time to be able to do the type of learning that we want to do. I like the connected math program because there is some *why* in there, some opportunities for inquiry. In another school I had 70 minute periods and sometimes longer, the scores went up considerably in one year. Connected math has its down falls but is real life situations. This is a VERY traditional town. Lots of pushback here for inquiry-based learning. Most of administration and teachers here are quite traditional. One section a day is what they think I should do. I need to do what is the best for the kids. I am the lone wolf. A couple of administrators are supportive.

Unlike Melissa, Sarah feels supported by her school administration in instituting her own beliefs about mathematics teaching and learning, although she adds that her administration is supportive only if she makes sure to cover a certain amount of material.

21st century learning, is where my belief system is, I support them but they construct real life mathematical situations. We are forced to cover certain pages of the book but that's not the vehicle I want to use, so they do

(administration) support my philosophy to use a different vehicle. Parents are very concerned kids should do math well in this community and the school as well, so parents support me as well.

Both Melissa and Sarah describes their time as a mathematics students in K – 12 as traditional and quite different from the experience they believe their students have in their own classrooms. Melissa notes:

In middle school, we had paperback book covers on our math books, we never learned like how I teach, wrote my integer rules on the front cover of the math book. I thought...I don't know why that works, but if you want me to change those two negative signs to a plus, I'll do it. I never knew what it meant; why it worked. School was hard for me so I blew it off. I wanted to be a social worker and work with kids. I took some trig and precalculus and it came very easy to me so teaching kids and math is what I decided to do in college. But I wanted to teach it better.

Beliefs About the Nature of Mathematics

Melissa and Sarah both feel that mathematics, as a subject, is relatively fixed. Sarah, for example, perceives mathematics as a relative static field with little new information, whereas mathematics education, she feels, has changed since her own math learning experiences.

Math has been around forever, so much has been discovered, a little is added at a time. Not too much stuff being added right now. We are getting an understanding of how the brain works which is changing how it is. Shouldn't be all just rule and drill and kill. They should be able to come up with it on their own and own it.

Melissa concurs, emphasizing that she thinks if mathematics would change more, this would have an effect on her school's surrounding community.

I work in an area has a very traditional view of mathematics. Parents think mathematics is a fixed set of rules and their kids should learn like they did. I agree that math is fixed but mathematics education has changed. My constructivist instructional style does not always go over well with parents.

Melissa's comments are typical of the challenges teachers face when attempting to implement instructional practice reform in the classroom (Battista, 1991; LIST).

Beliefs About Mathematics Learning

In her beliefs survey, Sarah strongly disagreed with the statement that in order to learn math, a student must be able to listen well, indicating additionally, in her beliefs interview, that active listening, active engagement and hands-on tasks are key. Melissa concurred, and both teachers felt students should solve problems in multiple ways and should be encouraged to discover mathematics on their own without explicit instruction. Melissa and Sarah both explained that students tend to do poorly when regularly placed in classroom situations involving following instructions and practicing skills as a focus. Rather, students should be given the chance to construct their own solutions to simple word problems before the teacher or another student demonstrates the solution. Strongly student-centered, Melissa and Sarah feel that students should construct the knowledge through hands-on tasks with teachers or students acting as classroom facilitators.

Beliefs About Mathematics Teaching

On her beliefs survey, Melissa indicated that she strongly believes that teachers should encourage students to find their own ways of solving mathematical problems even when it is an inefficient use of time. During this process, Melissa feels strongly that both teachers and students should share detailed approaches with each other. During their focused interviews, both Melissa and Sarah expressed that they felt they fostered such discussion daily. Melissa explained:

I watch and listen. I am there to guide. Sometimes I have to guide more than other times. Sometimes it seems like a disaster...I want them to like math, see that it can be fun. It's worth the time for them to think about things. I am pretty flexible. We work a lot together to try to figure things out. Sometimes I just have to switch gears.

Both teachers agreed that it was important to give students specific steps to help in problem solving, such as pointing out key words. Both agreed, however, that teachers should not always show how to solve problems before students attempt to solve problems on their own with support. Additionally, they felt strongly that students who have not solidified a math concept can successfully solve problems and should be encouraged to do so.

Although both teachers expressed strong beliefs, there was a struggle with balancing teaching practices and socio-constructivist beliefs, as Sarah explains:

I struggle with how much I should have them just practicing and using skills. No matter how much discovery learning is done, they can't get outside their comfort zone and they don't want to make a mistake. Even when I get kids that I know teachers are teaching that way with inquiry learning, they still are

just concerned about making a mistake. Some of them can handle it (what she refers to as inquiry learning). But not many. Just depends on the kid.

Sarah's Use of Specialized Content Knowledge in Practice

When I first walking into Sarah's classroom, I was taken aback by the walls that were covered with student-constructed posters involving the platonic solids, mock store "sale" posters demonstrating percent increase and decrease, a full wall covered with mathematics terms and so forth. What was most remarkable was the care that obviously went into each of the numerous mathematical works on the walls. On top of a shelf, Sarah showed me hundreds of other posters or other work that students had constructed to demonstrate core concepts in the curriculum.

On the first day of observations, students were presenting a project that they had prepared several class periods prior to my arrival. According to Sarah, the project involved Sarah giving students a mathematical property (commutative laws of addition and multiplication, associative property of addition and multiplication, and the distributive property, properties of equality). Sarah relayed that she then supported students in constructing their own lesson for the class focused on one of the fore mentioned properties. The project included constructing a full lesson, a homework assignment and an assessment. Students were allowed access to anything in the class to make the project a success. Using a hand-made poster, four students began a discussion of the distributive law, starting with some basic examples. Students demonstrated the distributive law across multiple representations. Sarah intervened only to probe students about the proper use of mathematical terms such as coefficient and reminding students that some mathematical properties also include operations.

Guys, it's not just a commutative property. Do we have a commutative property of subtraction? You need to tell me what operation you are talking about.

Sarah also stopped them if they did not explain a step.

Check if you get the same result if you do parenthesis first, remember the order of operations?

The students continue their presentation with various other participation from the audience. Mistakes are encouraged points of discussion. When the students finish, Sarah reminds students that they have now covered all of the properties and she tells them they will now be practicing them after discussion the concept of equivalence.

How we can check both sides of an equivalence is true? Yes, we can substitute values for variables. Now, remember our properties: distributive, associative, properties of equality, commutative and so forth. Why would I now talk about properties and what does it mean to be equivalent?

Remember that an equivalence may not look the same on either side but outcome of the evaluation on both sides are the same. We will use our properties now to demonstrate equivalence. I asked you to construct your own equivalences. Can someone volunteer to present their equivalence?

As students present their equivalences one by one, several mathematical situations arise, including when and how students can combine algebraic terms.

Why can't we combine $4x$ and $4y$? Good mistake to make. Underlines the common terms in different ways, visual. Let's start applying to rules by practicing.

Sarah then tells students that they are now going to now solve a problem and describe a situation with variables. During this, she says, we will practice our properties.

We are going to use variables to describe the relationship between rows and columns of chairs. We will describe r rows that are a across of chairs...variables can change depending on how many chairs we use Write a general equation. How is the number of chairs related to rows and columns of chairs? Come up with as many different equations to represent this as you can.

Sarah responds to student difficulty starting. Up to this point, according to Sarah, students have been solving systems of equations with one variable. I reflect on this as I observe, wondering how the students will be able to transition from the concept of solving an equation with one variable to constructing an equation that denotes a relationship between two or more variables. Sarah addresses the class again.

When you have a general equation describing the situation, how do we know we have an equivalence? Could plug in numbers to check. What numbers, class, do we avoid when plugging in numbers? Yes, we avoid plugging in the numbers zero and one.

It is unclear, even with further direct instruction from Sarah that students have come to a conclusion for the chairs problem. On Day 2 and 3 of observations, Sarah begins to discuss the use of exponents and exponent rules.

What does x to the third power mean? We can't "solve" it but what does it mean? Right the exponent and base, defines, explains what it means. Any number to the zero power is 1 except 0. 9 to the first power is just 9 because we just take the number and leave it by itself one time.

Sarah derives each of the rules. For example, to show that $a^x * a^y = a^{x+y}$, she starts with the example of $5^2 * 5^4$ and writes this expression as $(5 * 5) * (5 * 5 * 5 * 5)$ which she

notes is clearly 5^6 . Sarah then has the student develop their own example and derive a similar scenario. Derives each of the rules again for exponents in details, has students pick their own example to derive each rule, then practices each rule, moves on to next rule.

Sarah notes that in the expression a^x , the variables a, x can be anything. To show that a number to the zero power is 1 (Sarah notes the exception to this rule, when the base is 0), she shows students the following.

Since $\frac{5^3}{5^3} = \frac{5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = 1$ and according to the exponent rules we have used, $\frac{5^3}{5^3} = 5^0$.

Since $\frac{5^3}{5^3} = 1$ and $\frac{5^3}{5^3} = 5^0$, she then concludes that $5^0 = 1$.

Sarah gives students some time to think. She asks her students if they believe her. Sarah now responds to a student asking what 0^0 is.

Now, two takes on this mathematicians, it equals one or zero depending on the mathematician you ask, but we can't divide by 0 so we will say it's undefined. What happens when you put 0^0 or 1^0 in your calculator? Error, right?

Sarah concludes class by returning a recent quiz. Sarah gives students options for improving their quiz grades.

I have been grading your quizzes and I am not quite done. You get the better of the two grades, most of us are doing better, good improvements. A couple of you I will touch base with to see what we can do to improve your grade.

Sarah provided opportunities for students to engage in rich mathematical tasks. Sarah relied primarily on student participation and knowledge construction. This often resulted in students' presenting mathematical ideas that were less rich in mathematical language, often

included incorrect mathematical language and limited the students in exposure to the teachers' presentation of mathematical ideas as well as limited the amount of time spent on teacher-student discussions of the development of mathematical ideas and connections. Multiple representations were not a focus of Sarah's classroom instructional practices, limiting students' exposure and opportunities for connections between representations. Although Sarah clearly provided formal reasoning and justification for students' why questions, the quality of her responses varied. For example, in regards to explaining why $5^0 = 1$, Sarah presented an outline of a proof. In a responding to an additional student "why" question involving the equivalence of two expressions, she instructed students to "just plug in numbers" and "watch out for the special cases such as 0 and 1" which does not constitute a mathematically sound explanation.

Summary

Melissa and Debra clearly held similar beliefs about the teaching and learning of mathematics that were of a socio-constructivist in nature, clearly influencing their instructional practices. The beliefs-to-practice connection was consistent, despite several potential constraints discussed by Melissa and Debra. Both teachers provided students with numerous opportunities for potential exposure to rich mathematical language and discussion, connections between topics, and student construction of knowledge. However, the specialized content knowledge held by each teacher was relatively low in over the three-day observational period.

This particular case suggests a consistent, primarily socio-constructivist beliefs-to-practice connection at least opens opportunities for the use of specialized content knowledge in practice. Both Debra and Nancy were highly student-centered, typical of a socio-

constructivist teaching lens, yet these opportunities passed, limiting the quality of mathematics instruction. On the MKT survey, Debra and Nancy performed substantially better on core content knowledge tasks than specialized content knowledge tasks. MKT tasks involving core content knowledge are primarily procedural in nature. During the three-day observational period, these teachers were able to provide support for their students using core content knowledge. With limited specialized content knowledge, however, these understandings suggest what is well documented in the literature: mathematics knowledge for teaching matters, beyond core content knowledge, in assessing teacher quality (Shulman, 1987; Ball, 1990; Hill, 2007).

Consistency in the Beliefs-to-Practice Connection: Mixed Beliefs and an Illustration of Teacher Quality

Jake, Anna and Beth indicated beliefs in each of the belief categories of traditional and socio-constructivist at different times. Ultimately, these three teachers' beliefs were categorized as mixed. All teachers felt generally supported within their individual school climates in implementing his/her belief system and the beliefs-to-practice connection was generally consistent for all three teachers. Unlike the other three cases, Jake, Anna and Beth used specialized content knowledge often throughout the three-day observational period. Specialized content knowledge was regularly embedded in a variety of mathematical tasks. Similar to the case of Sarah and Melissa, a significant understanding gained from these three teachers is that the instructional practices aligning with a socio-constructivist view of teaching and learning appear to afford students the opportunity for exposure to specialized content knowledge. However, just as significant, was the vital role that a high level of SCK

plays in the delivery of quality instruction. Without this knowledge, in addition provide opportunities to for SCK exposure, the high-quality mathematics teaching observed would not have been.

Background and Setting

The town in which both Jake and Anna teach is a small town in the northeast, classified as rural/fringe³, and currently has a population of approximately 4300 people. The school population demographics school are similar to that of the local town, with a primarily White makeup (98%). The average family income for the town is approximately \$16,000 above the average family income in New Hampshire, with the district spends approximately one thousand dollars less than the national average per student per school year. Approximately 9% of the school population participated in the free/reduced lunch program in the district (National Center for Educational Statistics, 2011).

Beth teaches at a much larger middle school in comparison, with a population of 1,315 students. students (NCES, 2011). Median household income 76,980 per pupil expenditure \$10963 per pupil. 101 students eligible for free or reduced price lunch, primarily Caucasians.

Grade six has a total of 57 students in the 2011-2012 school year, with a total of 482 students in the Pre-K – 8 school. Both Jake and have a classroom of approximately twenty students. Jake and Anna both describe the school climate as one that is supportive and allowing for creativity in teaching. Jake and Anna both expressed that their school

³ The National Center for Educational Statistics' census defines rural, fringe as territory that is less than or equal to 5 miles from an urbanized, noting that rural areas comprise open country and settlements with fewer than 2,500 residents; areas designated as rural can have population densities as high as 999 per square mile or as low as 1 person per square mile. The word choice of *urbanized* should not be confused to describe Jake's surrounding school area as urban area (National Center for Educational Statistics, 2011).

environment is unique when compared to other school climates. Jake particularly feels the school is unique from other public schools in its supportive climate for teachers.

The school supports when I foster students asking why...why this is true. I can be creative with my teaching and allow them to construct their own knowledge but we always follow the state standards as well. I am completely supported in allowing students to explore connections and truly understand concepts but I am also supported when I make sure students have to practice and learn algorithms. Kids work together to build knowledge. We have a shared understanding of how to operate on integers...for example... because we developed the meaning of the integer operations together. Our assistant principal was an expert math teacher for a long time before she became an assistant principal. She was really talented, very supportive, all of our administration is supportive. I can definitely stick to my teaching beliefs here,

All three teachers believe that their earlier life experiences have influenced how they currently teaches. All three teachers described traditional math experiences growing up, and were disappointed. This was a driving force in choosing teaching as a career; all three teachers wanted to provide their students with a different, better experience than their own. Jake describes his middle school mathematics experiences.

Middle school was the worst time for me in math. It was nothing like this school and my teachers were nothing like me. They were nice...but classes and homework were super-dry. Very boring. We sat and listened and did problems, then we brought twenty-five problems that were usually boring. Sometimes we would check the odd problems, the teacher would go over the

even ones. I wanted a chance to talk about some of the odd ones, but that never happened, we checked solutions in the back of the book. Not all teachers were like this but most of them. You know, it was just like it is when we send kids to high school now...very traditional.

Anna also turns to her earlier K-12 mathematics experiences as influencing her current beliefs about teaching and learning mathematics.

I have always wanted to teach math even since 5th grade. I always loved math although because of the tracking system in my school, I was number six out of 5 and did not get to go into the advanced courses for middle school. I had to advocate for myself to get placed back into the advanced math courses. I want the same for my kids. No one should be number 6. All of my classes were traditional. Textbook. Problems. But I liked the subject of math so I stayed with it.

Both Jake and Anna Jake were undergraduate mathematics major and went on to complete masters degrees, focusing on both middle and secondary math education in a five-year teacher education program. Part of Jake's teacher education program was a requirement to explore teaching; Jake chose a placement in which he could be in a high school math classroom and explore teaching for a full semester under the supervision of a classroom teacher. Jake was able to participate fully with his supervising teacher, at times, taking over the class and was exposed to teaching Algebra 1 and 2, Geometry and Trigonometry. This experience solidified that he wanted to go into teaching, but because this particular high school experience was in what he considered to be a traditional high school math environment, he chose to then try middle school for his year-long internship experience,

fulfilling an additional program requirement. Jake felt this might help him determine if there would be less traditional-style teaching at the middle school level.

Both Jake and Anna's five-year teacher education program requirements included courses in both the education and mathematics departments. Both chose to take methods courses for both grades 6-8 and an additional methods course for grades 7-12. Jake and Anna's undergraduate years were mathematics intensive, including two semesters of calculus, multivariate calculus, differential equations, linear algebra, two courses in mathematical proof writing, history of mathematics, numbers and operations, two geometry courses, abstract algebra, analysis and statistical analysis. Additionally, both took the history of mathematics education, two additional mathematics education research courses, and a senior mathematics education seminar to discuss mathematics and mathematics education in more depth. Beth completed a similar program, but with a semester-long teaching internship. Beth, on the other hand, has completed work beyond her master's degree and is currently enrolled in a Ph.D. program in mathematics education and is currently working on her proposal. It is important to note that Beth has knowledge of the research done in the area of mathematics knowledge for teaching, including the MKT survey. Beth has had one experience of administering the MKT survey in the past but felt strongly that her experience would not effect her survey results or her participation in the study.

Jake and Beth relayed that they felt their methods courses were valuable in preparing them for their teaching. When Anna talked about her required math courses, she had mixed feelings as to their usefulness. She liked geometry and differential equations mainly because she appreciated the teachers, but she did not appreciate the other math requirements

Some teachers were just too abstract and could not unpack the information to me. Way too out there for me. I couldn't connect with that type of

teacher. I don't think I am at their level. I got more from my methods courses, the courses with mathematics and teaching information.

All teachers also completed a significant amount of other coursework in education including courses in educational structure and change, human development and learning, contemporary educational perspectives and educating exceptional learners. Lastly, Jake and Anna were required to complete a two-semester (one full one-year school year) internship in his final year of the program. The year-long internship was a culminating high point of what Jake and Anna both perceived as very comprehensive program. Jake felt strongly that the culminating experience of his teacher education program and specifically, the one-year internship, were major factors in his current beliefs and practices.

My teacher ed program had a big impact on my beliefs about teaching. I knew what I didn't want to do....like what I had at middle school growing up, but my teacher ed program gave me the research to back up what I wanted to do...when I entered the program, I didn't know I could teach another way other than how I was taught and we developed our own personal philosophies about teaching....we thought a lot about that in our program.

Jake and Anna finished their teacher education program as state-certified teachers for mathematics in grades 6-8 and also grades 6-8. In addition, Jake holds a current National Board of Professional Teaching Standards certification in mathematics. Beth holds several teacher certifications: math for grades 6-8, 7-12 and science for these levels as well.

Jake has taught now for two years as a grade 6 teacher in his current school but also taught grade 6 at a larger middle school. This is the extent of Jake's teaching experience.

Anna is in her 13th year of teaching 8th grade at the same school as Jake. Anna works closely

with Jake at the same school and although Jake teaches grade 6, they have regular interaction in weekly planning meetings specific to mathematics as well as daily in discussing teaching ideas. It is interesting to note that Jake's year-long supervised internship was done under Anna, at grade 8. Beth is in her 7th year of teaching and has held a variety of positions teaching gifted and talented (GT) full pull-out grade 8 classes as well as providing support in a resource room for students classified as GT. She has taught several years of science at the middle school level as well.

Both Jake and Anna actively searches for available professional development and participates when it is available. In the last year, Jake participated as an NCTM regional conference attendee, as well as a conference based on integrating mathematics and technology. Jake also worked on an on-line project involving instructional materials from Marilyn Burns, a nationally renowned mathematics educational consultant. Jake meets weekly with his district's vertical team which he describes as professional development which includes mathematics methods topics regularly.

Beliefs about the Nature of Mathematics

Jake indicated several strong beliefs regarding the nature of mathematics as well as the teaching and learning of mathematics content via both the beliefs survey and during his second interview. During the second interview, Jake discussed his beliefs about mathematics in detail.

I haven't thought about this in a while...I guess when I took math classes in high school I had more of a perception that answers were right or wrong. I would tell a student yes or no, explain and move on from there. Now I see the value in a partial solution and work with that. I don't value speed as

much as I use to. Here we tell students to do quality work...try solving things in different ways...I let them think about their understanding before we just dive in and show them. There is more to math than a bunch of steps here. We believe in using inquiry here, all the math teachers are basically on the same page. My closest held belief regarding mathematics as a subject is that problem solving is a life-skill. Solving problems is something that we do on some level every time we make a decision. I feel that mathematics is the best foundation for building problem solving ability.

Beliefs About Mathematics Teaching and Learning

When Anna discussed her beliefs about the nature of mathematics itself, she indicated that she felt mathematics was changing.

Mathematics is definitely changing. I don't keep up on current research in math but every-so-often I hear about a major advance or major theorem that took years to finally prove. I think mathematics is so important in developing reasoning which they will use for the rest of their lives. I think students think math is super-static in K – 12. I guess it seems that way, although things have changed as to how we actually teach it and what we know about how students learn it.

, in the beliefs survey, Jake indicated he strongly believed (a five-point option on a 5-point Likert scale) that math should be taught in such a way that students can discover their own connections. Additionally, Jake indicated that he strongly believed that teachers should guide the construction process of mathematical knowledge as students learn, rather than

transmit discrete knowledge. These two strongly held beliefs are indicative as a socio-constructivist view of teaching mathematics, where students are given the opportunity to learn mathematics by personally and socially constructing mathematical knowledge (Cobb & Yackel, 1991). Teaching strategies typically noted in the literature tied to a socio-constructivist view of teaching are supporting generative learning, reflection and exploratory processes with an emphasis on understanding (Handal, 2003; Stipek; 2005; Wood, Cobb & Yackel, 1991).

Another strongly held belief indicated by Jake was that students should understand procedures before just using them. This belief was indicated multiple times in similar belief survey items and during Jake's focused interview. Jake is confident that students can discover math concepts on their own without explicit instruction. These two particular beliefs fit within a belief system where aspects of socio-constructivism prevail (Simon, 1995). The literature reveals social-constructivist views of teaching fit well with the belief that teachers should allow students to devise solutions to simple word problems before teacher demonstrates the solution, an additional belief that Jake indicated he held strongly.

Jake indicated that teachers should share detailed approaches to solving problems as a form of scaffolding when students are first exposed or struggling with a concept. Jake also felt strongly that students should work together to talk about mathematics, share ideas and come to a mutual understanding. The nature of these beliefs, taken together indicate that although Jake believes teachers should allow the opportunity for student autonomy and opportunities to construct their own knowledge, this teaching approach must be tailored to include teacher facilitation and scaffolding to ensure students move forward in conceptual understanding. For example, although Jake indicated that he agreed teachers should encourage students to find their own ways of solving mathematical problems even when it is

an inefficient use of their time, he also felt that teachers should provide support to those having difficulties solving word problems with guidance and possibly additional instruction. Jake clearly believed that he was there to facilitate learning and support students as needed throughout instruction.

Beswick (2005), who after doing an extensive review in the area of teachers' mathematics beliefs as well as her own extensive work in the area of teachers' mathematics beliefs and their practices, surmised that teachers holding beliefs that are primarily Platonist also hold particular beliefs about the teaching and learning of mathematics that fall into the category of social-constructivism. For example, for the beliefs that have been discussed thus far, we might surmise that Jake could be classified most accurately as a Platonist with beliefs in regards to beliefs about mathematics. Additionally, Jake's beliefs would fall into the realm of social construction of knowledge with a focus on understanding, learning.

On the other hand, when inspecting Jake's beliefs more closely and obtaining further information from Jake, there were some beliefs that could be classified as outside of this framework. For example, although Jake indicated in both the beliefs survey and focused survey that he valued some traditional math teaching practices such as practicing skills, Jake indicated that he valued traditional math teaching practices for other reasons.

Kids have to go on to 8th grade and then high school in a world where things are traditional at high school. We have to expose them to that often enough to prepare them for that, unfortunately.

Practice is valued by Jake. He also indicated that practice should not be a focus or a sole way of learning mathematics. Jake indicated in the focused survey that he believes there should be a balance of conceptual understanding and computational proficiency. These

different belief categories demonstrated the beginning of a myriad of complexities in teachers' belief systems in this study.

Jake's Use of Specialized Content Knowledge in Practice

I now take a look at Jake and his use of specialized content knowledge in his classroom. As I proceed, I will preliminarily reflect upon Jake's indicated beliefs as he uses specialized content knowledge in the classroom in different teaching episodes surrounding particular mathematical tasks of teaching. I grapple with several questions. How does Jake use specialized content knowledge in practice? What more can we learn about the complex system of beliefs Jake has indicated through observing Jake's use of SCK in practice? Can we provide some explanation as inconsistencies between beliefs and practice arise? What can we learn about the relationship between beliefs and SCK use?

Jake's three classroom observations were done consecutively on a Tuesday, Wednesday, Thursday, within Jake's regular classroom, within one week of Jake completing both the MKT and beliefs surveys. When asked about the general area of content for the classroom observations, Jake relayed that he would be talking about "solving equations." Jake's class meets for about fifty minutes at different class times each day, beginning at 10:30 on the first day, then about a half-hour later on Wednesday and began at 11 a.m. on Thursday. Students sat in groups of four and the four desks were set up to have two students directly facing the other two students. Although this room was used for other purposes, such as social studies and other subjects throughout the day, Jake's walls were full of a variety of resources specific to mathematics and mathematics literature on the wall and shelves, three computers, two whiteboards and one smart-board with an equation balance ready to go. The wall contained a PEMDAS poster.

On Day 1 of observations, Jake began the class with a 5-minute, 10 question warm-up activity on the smart-board. While students began working on a problem involving the term *regular polygon*, Jake realized that students were not clear on the definition of a polygon. Jake found several examples to help students build the definition. Throughout this process, Jake was critiquing the definitions that students were offering, encouraging mistakes and adding more examples to help illustrate different properties of regular polygons. Jake also encouraged students to use mathematical terminology such as polygon and other geometric terms, and modeled this in his own presentation of material.

Regular or not regular – this is a geometric term – what do you think...let's think about what is regular? When you think of a hexagon, pentagon, what is a regular one? Can we think of one that is not regular?

Jake writes a triangle, square, pentagon and hexagon on the smart board.

I am making regular polygons. What do we notice? There is more than one quality.

Jake responds to a student who claims that a regular polygon is a polygon where the sides are all the same length. To demonstrate that the triangle Jake drawn had sides of the same length, he rotated the triangle multiple times, each time by 60 degrees.

Are you sure they are the same length? Do you need proof? You can trust me because I am a math teacher? Let's look at the symmetries of the equilateral triangle, square, pentagon and hexagon. Do you need more proof? What other properties are there? Can a side be curved or straight? What do you think our definition of a polygon is?

On Day 2 of observations, during another five-minute warm-up whole-class warm-up, a question was posed on the smart board requesting that students compute $6 - - 4$. After noticing that students were hesitant to start this problem, Jake poses questions to the class.

Can anyone tell me what this expression means? Are we adding or subtracting? Ok, so if we are subtracting, is the first number positive? What about the other number? What is the opposite of negative 4?

Jake regularly probed students with productive statements during class whether the class was involved in whole-class discussion, groups or working individually. Often his probes included glimpses of unpacked mathematics content knowledge. In essence, Jake worked with student ideas by unpacking content so that students could access building blocks for construction. Much of this construction took place in groups due to the way students were seated as well as Jake's frequent daily encouragement for students to discuss their ideas.

After decompressing the expression, Jake asked the students to look at the number line on their desks, a traditional number line from -10 to 20, marked with the integers only. Additionally, Jake asked students to also look at the board, which had the initial representation of the problem and helped students make the connection to an additional number line that indicated positive numbers in blue and negative numbers in red. He quickly distributes manipulative that represent the integers as colored in the same manner as the smart board number line. Jake watched a student use the manipulatives to demonstrate an attempt to match the six individual positive chips with and told the student immediately that he was correct. He told the rest of the class to take their time, encouraging students to discuss the problem amongst them. He aided students in making connections between the chips and the abstract operations on the integers.

Math is not about speed. In fact, speed can increase errors. Remember we talked about taking our time to understand and check each step so that we prove to ourselves that we are right. Part of this process is making mistakes so don't be surprised if that happens. Explain your thinking to each other, even if you are not sure if you understood why the answer is 10 yet.

Jake then connected the problem to another problem the class had done last year and during the last month. In responding to student *why* questions about operations on integers, Jake reminds students of a story his middle school students develop over their middle school years used to aid students in conceptualizing operations on the integers.

Remember our mail man we used last year and started again this year?

Remember what was good or bad when he delivered mail? Subtracting -5 is like taking away a bill, taking away debt, taking away that you owe someone \$5. Is that good or bad? If the postman bring you some credit or cash, is that good? Like adding $+6$, for example?

Jake then selected another example, and offered another problem similar to the previous problem but with larger values that can sometimes be more problematic, that forced the students to have to match a new number of pairs with their manipulatives, $12 - -29$. Later, Jake asks students if $4-x$ is the same as $4(-x)$.

Is the operation of multiplication hidden in of these examples? To see if these two expressions are equivalent, we could substitute numbers in for x in each expression. Let's try it out, plug in a number for x . How do you know it's always true if it is? We need to remember what a coefficient is. Try it out, if $-x = 4$, notice that x and $-x$ are opposites. x is the opposite of $-x$.

Do we remember order of operations? Will this help us here?

At this point, Jake diverges in a review of order of operations using the pyramid in Figure X above. This pyramid is noteworthy because unlike the PEMDAS algorithm that typically leads to difficulties in understanding that same-preference operations are performed from left to right, the pyramid levels indicate that same-preference operations are on the same level, with a left to right arrow. Jake specifically chooses examples that address same-precedence operation confusion such as presenting the problem for students to compute $2 - 5 + 6$.

Back to equating the expressions on the board, encouraging students to find a value for x and see if the equality holds. Responding to students' attempting to disprove $4-x = 4(-x)$, for all values of x . Students were then split up into groups different than their seating arrangement groups and were given five different options to work on five different centers, each of which contained different tasks involving working with integers (and also solving equations). One center involved a worksheet of word problems. Students were again reminded to talk to each other and work together. It is important to note that during any independent or group work during the three observational periods, Jake was present continually engaging students in mathematical discussions and encouraging the use of accurate mathematical language and notation. In responding to concerns about having to finish a worksheet including these topics, that was perceived as lengthy, Jake responded by saying,

Let's try to do at least 3 or 4. It is o.k. if you do not finish. Let's just make sure that the work we do is quality work. Talk to each other to see if you can come up with a plan or come to me if your plan isn't working.

Jake often came up with examples spontaneously to make a specific point such as how to approach equations of one variable with terms involving x on both sides of the equation.

Jake chose several examples that varied in difficulty to aid students in understanding steps useful in moving forward. Jake intentionally chose examples that cause common errors and he illustrated these common errors as he had students guide him through finding a solution. His examples also included cases where a solution was not possible or the case of infinitely many solutions.

Jake regularly throughout his three days modeled the use of fluent use of precise mathematics terminology and ideas, although Jake used the word “opposite” frequently in all three observation days as his description of a number and its additive inverse when he directed students to look at the number line – for example, the numbers 4 and -4, Jake referred to as opposites. Although this creates a visual representation for the place of both numeric values on the number lines, it did not expose students to the terminology of the property of additive inverse, a key mathematical term in middle school (CITE). This did not overshadow the overall quality of instruction in all three days. In particular, it did not overshadow Jake’s regular use of specialized content knowledge in instructional practices and the opportunities for student to engage in mathematical tasks involving the use of SCK. I now turn to Anna, that teaches at the same school as Jake, but teaches 8th grade.

Anna’s Use of SCK in Practice

Although Anna indicated that she held beliefs that would primarily be considered socio-constructivist, she stated that she has found herself, over 13 years of teaching, moving back towards a traditional approach or a mixture of the two philosophies, at least for some students.

Sometimes I feel bad because some kids need the traditional approach. I

think I am making a swing to more traditional because inquiry for some kids

is banging their heads against the wall over and over. Sometimes one student I have needs to crank out the problems, then step back and think about where it comes from. From observations of kids over the years. My own teachers taught the same way, but I connected with them depending on who they were as a person. Sometimes too many choices for kids can get in the way for kids. Overwhelming. Project stuff has to start really young if we wanted to do that all the time.

Being an 8th grade teacher, with students close to their high school years, Anna feels that preparing inquiry-based learning for all students all of the time can put students at a disadvantage for high school.

We have high school teachers that say you have 2 minutes to take this quiz but since I am sending kids to 9th grade I would do them a disservice purely to do projects-based. In first half of 8th, we do not do timed tests, second half, we do timed tests. To help them transition. It's not my only goal to get them ready for high school, but it's a big piece. It's a little sad reality. High schools are primarily traditional and our population goes to public high schools and private schools.

Not surprisingly, although Anna appreciates the use of manipulatives, she acknowledges at times that she focuses more on traditional styles that prepare students for high school, which includes "brute force problem solving" on paper. She has also found over her 13 years of teaching that students in 8th grade tend to find the manipulatives to be a waste of their intellectual time.

We could use film canisters for variables in equations. I don't really do this as much anymore. Manipulatives do help, hands on equations, for factoring

and multiplying – and that x cubed is not $3x$ and other misconceptions. 8th graders are like, manipulatives, why do we need to do manipulatives, especially the students who are doing well, they think it slows them down... but I tell them... it's another skill to see it visually and I value this skill so you have to be able to show me that you can see it. If I found a solution, why do I have to find a solution another way? I want them to value multiple solution pathways and multiple representations so I insist they use manipulatives at least for factoring and multiplying.

Whether students learn to value multiple solution pathways and multiple representations due to these experiences is not clear to Anna.

Anna believes that it's important for students to know why the rules in mathematics work. For students to understand properties, such as the distributive property or properties of equality, she believes students should demonstrate for themselves that a statement or property is always true.

We plug in numbers, they get it. We plug in all types of numbers, not just a few. Special cases like 0 and 1. The more numbers they plug in, the more likely they are to see the pattern and be able to generalize to a property that is always true.

Anna believes strongly that students should be shown atypical examples and non-examples often. In the following MKT released task (cite), Anna discusses that the below task is one that she uses in her teaching.

Visually a good example because it is not a square, which is the typical presentation from teachers and textbook. This clearly shows the distributive property, good figure.

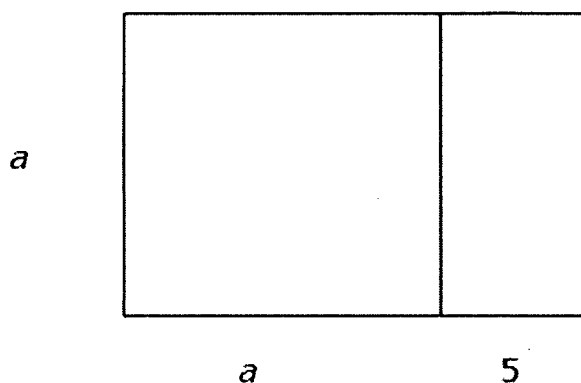


FIGURE 14. Julie's Shape Discussion

Summary

Jake primarily indicated beliefs of a problem-solving nature when discussing the subject of mathematics. Across data sources, Jake consistently indicated a learner-centered view of how students learn mathematics, and a problem-solving view of mathematics teaching, with exception that Jake indicated that he believed that practice with mathematical skills is important and should be a regular occurrence. Although practice was encouraged, it was not a focus of Jake's classroom activities. The majority of Jake's classroom activities involved providing opportunities for students to participate in a variety of quality mathematics instructional tasks, with Jake as the facilitator.

During Jake's 3-day observational period, Jake regularly used specialized content knowledge as a way to present students with unpacked mathematical knowledge accessible to students who then could use these building blocks to construct ideas. Jake presented students with multiple situations where students were allowed choice and exploration. Jake spent a significant amount of classroom time introducing different examples that included accurate use of mathematical language, yet language that was accessible to a learner via the use of multiple representations of such knowledge. He regularly made connections between topics by providing students opportunities for such connections to occur. He picked

examples that addressed common student misconceptions (use $-4(x)$) Jake was able to present different representations of the same example to address student misunderstandings. Identifying the errors in reasoning (when he walked around the room). He was able to respond to a student to determine if the student's method would always work in the case of any two integers. Recognized easier and harder numbers to use in examples. Jake offered reasons why student solutions work or why he chose certain steps. Jake had to know that the manipulatives he chose, how to use them to get students to understand and connect to the balance. When to use or not special cases 0, 1. Examples and non examples when building a definition.

Jake's practices were generally consistent with his expressed beliefs. Jake did not express constraints on his enacting his beliefs related to his school climate or surrounding community. In fact, Jake relayed that he felt that he was supported fully within his school climate in implementing his belief system. Jake articulated his beliefs about the teaching and learning at length; indicating the roots of his beliefs primarily developed in his teacher education program during a course in which Jake constructed, wrote about and reflected upon his beliefs after completing coursework in mathematics education and what Jake refers to as "what the current best practices and philosophies are out there."

A significant understanding gained from Jake's case study is that the instructional methods that are indicated in the literature that generally align with Jake's socio-constructivist views of teaching and learning appeared to afford students the opportunity for exposure to specialized content knowledge through mathematical tasks listed above (cite). This is not about whether some teacher beliefs are "better" in the sense that teachers should use these instructional practices. Rather, the understanding drawn is that *expose* students to SCK if instructional methods (Table X) are used that afforded these opportunities for

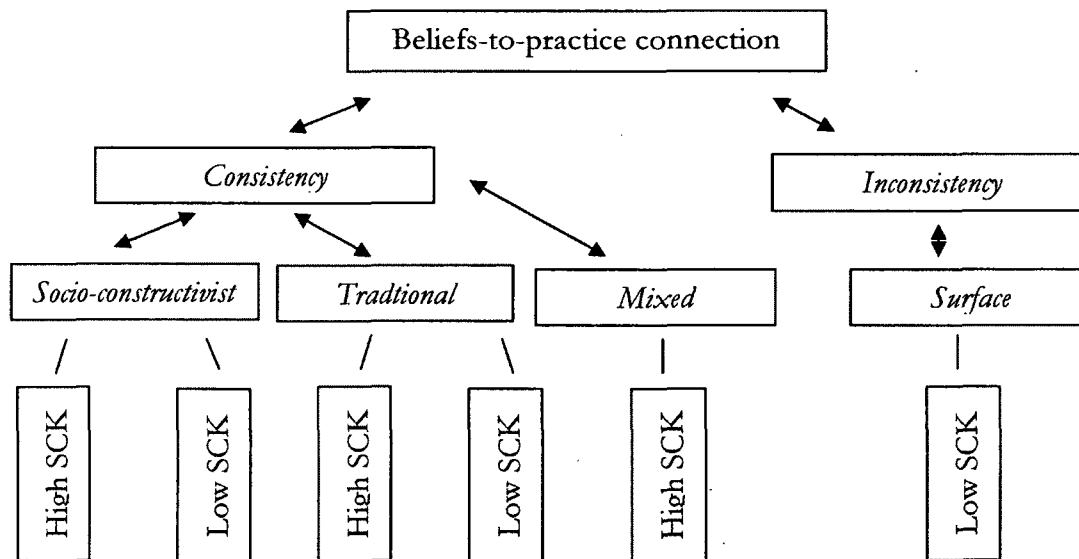
students. Jake's consistent use of tasks such as responding to students' "why" questions, linking and connecting multiple mathematical representations, choosing and developing accurate definitions, infiltrating tasks with mathematical language and notion, allowed students the exposure to specialized content knowledge.

Jake acted as a facilitator in this process of exposing students to SCK, as well as guiding the construction of the SCK building blocks as the building of knowledge occurred. Clearly, this is possible only when a teacher holds specialized content knowledge, which is closely tied to core content knowledge, as well as the other constructs in the MKT model (knowledge of content and student, knowledge of curriculum, and so forth). Jake confirmed holding such knowledge, using the knowledge in his classroom observations as well as demonstrated a high score on the MKT survey, a survey which has been shown to correlate with quality mathematics teaching.

Jake, Anna and Beth consistently indicated beliefs of a socio-constructivist nature. For example, Jake indicated strongly, in multiple contexts, that students should be given the opportunity to learn mathematics by engaging in a variety of mathematical tasks to socially construct mathematical knowledge. For all three teachers, the beliefs-to-practice connection was generally consistent. Only Beth expressed constraints on enacting her beliefs related to his school climate, yet indicated that she did not feel these constraints significantly impacted her practice. In fact, all teachers felt supported fully within their individual school climates in implementing his/her belief system. Each of the three teachers pointed to their teacher education programs as the roots of his/her beliefs primarily developed during a teacher education program course involving the exploring, creation and reflection of a teaching philosophy statement. Jake, Anna and Beth appeared to have the recipe for quality mathematics instruction.

For clarity purposes, I again offer the following organizational chart to explain the understandings that have emerged:

FIGURE 15. Overview of Results



As a result of the analysis of beliefs (determined by the beliefs survey along with clarifications from the beliefs interview), beliefs generally fell into four categories: surface beliefs, primarily traditional beliefs, primarily socio-constructivist beliefs, and lastly, primarily a mix of traditional and socio-constructivist beliefs, with the classification depending heavily upon the consistency of the beliefs-to-practice connection. Within this framework, teachers varied in their MKT (including CCK and SCK) level and SCK use in the classroom as discussed above. I turn now to Chapter 5, where I decompose the above structure, providing insights and interpretation of understandings found here.

CHAPTER 5

DISCUSSION AND INTERPRETATIONS

In Part I of this two-part study, confirmatory factor analysis was performed on Mathematics Knowledge for Teaching (MKT) survey data (Hill, 2007) resulting in multiple descriptive categories of the different types of mathematics knowledge teachers hold. In particular, the survey items proposed to measure common content knowledge or specialized content knowledge reflected in a conceptual knowledge or procedural knowledge base, not entirely mutually exclusive. In Part II of the study, each of the four general cases provided a unique picture as to how common and specialized content knowledge was used in the classroom, particularly in relation to the enactment (or not) of teacher beliefs. A teacher's belief system did act as a "filter" in how CCK and SCK were used and contextual factors, such as school culture, certainly played a role in whether certain beliefs were enacted. Ultimately, a teacher's beliefs about mathematics teaching practice appeared to have as much of a role in determining quality mathematics instruction as the level of specialized content knowledge teachers hold.

We do not yet fully understand the model of mathematics teacher knowledge that practicing teachers hold, yet the importance of such knowledge is clear. It is urgent that teachers hold this knowledge and that teacher preparation course work that reflects not only mathematics knowledge for teaching, but the importance of the relationship between teacher beliefs, mathematics knowledge for teaching, and the relationship to actual teaching practice. This last idea points to the importance of ending the typical divide that exists between mathematics content and pedagogy in teacher education. Most notable, both parts of the study pointed to major problems in equity in student access to quality mathematics

instruction, similar to Hill's earlier findings in 2007, where she noted significant variance in mathematical teacher knowledge across the United States.

Before the details of the above ideas are discussed, I provide a review of the participants and the varying backgrounds they brought with them to the study, as well as the general cases resulting from Part II for the reader to reference along with the Chapter 5 discussion. The below table includes the range of educational background, mathematics and mathematics education courses, certification and teaching experiences. Each color indicates each of the four general cases from Part II of the study (see table below).

TABLE 25. Participant Background Information

	Teacher Preparation	Math Courses	Methods Courses	Math Certification	Teaching Experience
Robert, Grade 6	24 M.Ed. Credits	6	2	State MS	MS, 11 years
David, Grade 5/6	M.Ed. Elementary Education, 1 semester internship	1	1	None	LS/MS, 3 years
Julie, Grade 7	B.S. Architecture	4	0	None	MS, 5 years
Debra, Grade 8	M.Ed. Secondary Education 1-semester internship	9	2	State MS/HS	MS 7 years, HS, 15 years
Nancy, Grade 7	M.Ed. On-line education program	6	1	State HS	MS, 1 year
Sarah, Grade 7	B.S. Mathematics Middle School Teaching, 1-semester internship	8	1	State MS	MS, 7 years
Melissa, Grade 7	M.Ed. Elementary Education, 1 semester internship	8	5	State MS	MS, 3 years

Table X. Overview of teachers' educational experience and credentials.

From Part II of the study, cross-case synthesis (Yin, 2009) was used over the ten individual teachers informing the general themes that emerged as indicated here, with teachers varying in SCK knowledge:

- Inconsistent Beliefs-to-Practice Connection: Surface Beliefs (Robert, David & Julie)
- Consistent Beliefs-to-Practice Connection: Traditional Beliefs (Debra & Nancy)
- Consistent Beliefs-to-Practice Connection: Socio-constructivist Beliefs (Sarah & Melissa)
- Consistent Beliefs-to-Practice Connection: Mixed Beliefs (Jake, Anna & Beth)

Specialized Content Knowledge as the Learner's Building Blocks

This study is one of many that describe the significant role of specialized content knowledge in quality mathematics instruction. Without the presence of SCK, instruction is generally focused on common content knowledge which was typically delivered via teaching methods such as teaching by telling. Procedural knowledge is clearly important. However, common content knowledge without the use of specialized content knowledge is similar to a chef with a recipe and no craft in cooking. For example, when Debra responded to a student inquiring about the first step needed to solve an equation with variables on both

sides, she said, “We move all the x’s to one side and we move all the numbers to the other side. That just makes things simpler. It’s what you do each time. Variables to the left, numbers to the right.” When Jake responded to a similar question, he responded by asking the student where the student thought the first step should be. Next, Jake encouraged the student’s choice and guided the student in moving forward with the student’s own solution method. Jake acknowledged the student when the student indicated that she realized, under his facilitation and through her own thinking, that she could work toward isolating the variable using any step that did not violate properties of equality. In discussion with students, Jake often was able to decipher middle school math student language, pinpoint what a particular student was struggling with, and provide the necessary building block in the form of a representation, and often more than one, the student needed to move forward. Jake clearly was able to draw from both his conceptual and procedural knowledge, whether it be specific to teaching or not. He demonstrated a high level of MKT that along with his belief system played out as a high quality instruction. Of the questions in Part I of the study that clearly did measure SCK, Jake had a perfect score.

When Sarah opened class discussions by asking potentially productive mathematical questions, when engaging in discussions with students, her language was often unclear due to imprecise mathematical language. When Julie attempted to provide explanations to students with questions, her answers were typically of the form, *we just do this because this is how it works*, citing procedural knowledge in the process. When Richard used different representations during instruction, some representations did not accurately represent the mathematical problem at hand, and at other times, although Richard could explain how students should use the representation, it was unclear *why* it was being used or how it connected to the mathematical concept under discussion. Thus, the lack of specialized content knowledge

was a major indicator of quality instruction or lack thereof in this study. Additionally, Sarah, Julie and Richard scored in the mid-range on common content items, yet most SCK items were missed.

The Development and Nature of Beliefs

Interestingly, all ten teachers referenced their negative experiences in their own K-12 learning experiences and expressed a need to be a different teacher than they each experienced. This need to “do better” for their students was masked. This was partially due to some teachers expressing a need for their students to experience a traditional mathematics classroom experience. For example, two teachers indicated that their own experiences were negative, yet they believed they learned mathematics as a result and hence if it “worked for them,” then it should also work for their own students. Most of the participants indicated a value of skills and practice, expressing that exposure to a variety of types of problems involving a basic concept allows for exposure to multiple examples and the building of the concept. It is interesting to note that all participants felt the need to prepare their students with traditional methods. According to Dana, for example, she felt that students would be facing these teaching methods in their high school and college experiences and hence they should be exposed to such practices now in order to be fully prepared. Teachers were preparing students for future college experiences in mathematics or related fields, some of which, ironically, are advocating for mathematics reform that generally does not fall within such practices. Teachers who scored highly on items found to factor into the SCK (primarily conceptual knowledge) were also teachers who tended to enact socio-constructivist beliefs consistently within their classroom and voiced in a number of ways, during interviews, of the importance of SCK.

Some participants also indicated that their development of beliefs was again influenced after their K-12 experiences when they enrolled in their teacher education program. These participants, most notably Sarah, Melissa, Jake, Dana and Beth, pointed to the significance of teacher education course that exposed them to new and alternative teaching and learning philosophies, and encouraged their reflection on their beliefs about teaching and learning. These five teachers highlighted this experience as contributing to major changes in their educational philosophy that they felt had significantly impacted their teaching practices.

Beliefs clearly influenced daily practice. Surface beliefs were hard to gauge and the belief-to-practice connection was not surprisingly unclear or inconsistent. Teachers with primarily socio-constructivist views of teaching and learning generally provided their students with opportunities for specialized content knowledge use. Unfortunately, if a teacher did not also hold such knowledge, opportunities were lost. Teachers with high levels of specialized content knowledge but held beliefs that were either constrained or did not lend themselves to opening opportunities for SCK use were also unable to provide students with access to SCK in practice. In essence, SCK was not enough; what teachers believed about mathematics instruction mattered just as much.

Some teachers indicated a continuation of the development of their beliefs, as ongoing reflections prompted change in how they thought about their practice. Specifically, the nature of the beliefs/practice relationship seems to be dialectical in nature (Wood et al., 1991), both beliefs influencing practice and practice influencing beliefs. For example, Dana indicated that although she had been teaching for several years from a socio-constructivist perspective, she relayed that she values some traditional mathematics practices due to her experience over the years. Specifically, she recounted how some students simply cannot

handle the inquiry-based and construction of knowledge. According to Dana, these students need something different that is more aligned with traditional mathematics teaching.

Debra's students' positive results on standardized testing, which she in turn believes is influencing her support from administration, is likely reinforcing her beliefs about her teaching practices, signifying the dialectical nature of the beliefs-to-practice connection.

The Role of Context

The role of current context became clear at different points during the study. One example was during structured interviews. Due to the structured nature of the interview as well as the setting of a researcher in the same room observing only participant, in conjunction with what most participants referred to as "a time of caution" when evaluations occur, teachers appeared quite uncomfortable with the structured MKT task interview. *The preceding does not make sense and I couldn't figure out what you were trying to say.* All teachers made comments regarding the current state of political affairs for teaching and the focus of teacher evaluation. Teachers regularly interrupted the structured interview to reaffirm that the author's role was not to evaluate. Nevertheless, this finding was itself important. Clearly, teachers are concerned about their current job status and the role that evaluations might play.

There were a variety of contextual factors that influenced the belief-to-practice connection. Robert and Debra both felt pressures of time, school climate and standardized testing to be constraining their beliefs implementation. All three eighth grade teachers expressed the concern about preparing middle school students for high school mathematics. Specifically, two of these teachers indicated making significant changes in instructional methods that were not aligned with the current belief system to expose students to

traditional teaching methods in preparation for high school. Included in this discussion is their hesitancy to use different representations and instead focus on skill and practice. Several teachers pointed to how parental influences pressure them to stick with traditional teaching practices.

Debra and Anna both expressed concern about using an area problem as a visual aid for teaching the distributive law. Anna indicated that she felt the exercise would be useful for students because she felt it was an atypical example of a representation which uses the distributive law. Debra indicated that she would not use that to teach the distributive law, below.

“They never get this. When we did integrated and let’s get all these little manipulatives. I could get them to do this but take this away, they saw no relationship between the manipulatives and the abstraction of the expression of the distributive law. Lower level kids didn’t get it. Upper level kids didn’t appreciate it or get the connections. The hands on equations, lower level group, helps a group of kids when you pull them away from the manipulatives to help them solve equations. Still, the Algebra kids don’t want any part of them. I know how to do it, why do I have to use this to get what I am doing again?”

Dichotomies

Emphasis of Conceptual Knowledge Versus Skills/Practice

No one can acquire conceptual understanding, problem solving skills, or basic skills individually; rather, they go hand in hand. This false dichotomy of skills and understanding impedes efforts to improve mathematics education. The dichotomy has arisen “from the common misconception that the demand for precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding. The truth is that, in mathematics, skills and understandings are completely intertwined” (Wu, 1999, p. 1). Some mathematics educators assert the teaching of standard algorithms are not helpful and hinder children’s development of numerical reasoning (Carpenter, Franke,

Jacobs, Fennema & Empson, 1998; Kamii & Dominick, 1998; Mack, 1990) while others take the stance that learning standard algorithms is only harmful if they are not taught properly (Ma, 1999; Wu, 1999). “The importance of automaticity (in basic computational skills) becomes apparent when it is absent” (Wong & Evans, 2007, p. 91). All agree that both are valuable; it is the location and timing and how of the teaching of basic skills in the teaching and learning of mathematics in the middle school grades that is still up for debate. All participants pointed to this dichotomy at some point in their focused interview.

Interestingly, this dichotomy was evident in the MKT survey items during factor analysis as well. When I first took the survey, my opinion was that the questions tended to pull from pure conceptual or procedural knowledge in mathematics knowledge but not necessarily knowledge specific to teaching. After factor analysis and reflecting upon the specific Part I findings, I hypothesize that the MKT items generally fall into pure conceptual or procedural knowledge in mathematics knowledge but not necessarily knowledge specific to teaching. Very few items fell into the true definition of SCK. Items that did seem to fit the SCK definition did capture a snapshot of classroom use of SCK within a typical classroom moment. These items were extremely valuable in differentiating between participants in Part II and would serve as important professional development tools in the development of SCK in teacher education programs and professional development in general. Capturing more moments of authentic SCK use in the classroom would have major implications for teacher education and the overall improvement in SCK knowledge in mathematics teachers.

Socio-constructivist Versus Traditional Beliefs

Although the review of the literature on teacher beliefs specifically within the domain of mathematics revealed such a dichotomy, I consciously constructed opportunities for participants to explore their beliefs about teaching and learning that were outside of this framework. Despite these opportunities, participants generally indicated beliefs of either category. All participants had some level of awareness of these two worlds of beliefs as the two acceptable categories and used the two categories as reference points in discussion. All participants had difficulty discussing one school of thought or the other. As Anna notes, “Well, I primarily have constructivist beliefs, I think. But practice *is* important. I feel like I can’t say those two things together. Practice is a bad word these days.”

Other Dichotomies

Teacher education programs in particular require future mathematics teachers to take both mathematics and methods courses often taught within two different departments which frequently results in teacher beliefs that reflect a dichotomy between mathematics and mathematics teaching (Potari, 2001). Another dichotomy that arose in the interview process was that of focusing teaching on abstraction versus the use of manipulatives. Anna indicated times in her teaching where she felt the use of manipulatives aligned with her socio-constructivist beliefs about teaching and learning, whereas other times highlighted the need for students to experience abstraction and what she refers to as “higher-level” math thinking. She also indicated that as students move forward in their education, they transition not only toward a more traditional learning environment but one of less hands-on manipulatives and more abstraction. She also noted that her students who are “quick at math” view manipulatives as tools for “slower math kids” and “a waste of time” and a hinderance in

getting quick answers. She stated that the higher the grade level, the more resistant “the bright kids” were to using manipulatives.

The Role of Teacher Education

As expected, understanding teachers’ beliefs and the belief-to-practice connection is a highly complex endeavor. Laying the foundation of multiple data sources, as well as investigating knowledge use within the domain of mathematics addressed some of the complexity. Viewing teachers’ beliefs in multiple forums allowed for some clarification of inconsistencies. If clarification was not possible, I delved further into why inconsistencies occurred through questioning factors that typically influence the formation of beliefs (teacher preparation, a teacher’s own K-12 educational experience, social or professional context).

Perhaps it is unrealistic to expect teacher education programs to have a substantial influence on teachers’ beliefs. However, it is quite clear from this study that participants who completed teacher education programs which included what participants felt were substantial conversations, reflection and development of beliefs about teaching and learning made statements indicating that they had a well-thought out belief system, which included the ability to define, plan and deliver practices that reflected their beliefs. They highlighted the importance of current and ongoing reflection about their beliefs and practices on a regular basis. It is well established that teachers’ beliefs influence teaching practices, and prospective teachers would benefit from opportunities to discuss and reflect upon their beliefs, including discussions about how beliefs might influence practice.

There is long history of a need for the integration of content and pedagogy within the same course in the preparation of future mathematics teachers. Ironically despite little disagreement over the need for additional courses such as these, there is very little

movement nationally towards this goal. Over the last 20 years, very few programs have added additional requirements that would substantially change the mathematics preparation level of future mathematics teachers, specifically at the elementary school level.

Study Limitations

The reliability and validity of each data collection tool in Part II was discussed, as well as limitations to the factor analysis performed in Part I of the study. Although several data collection tools in Part II of the study lacked proper psychometric properties and substantial procedures to address reliability and validity, due to time and resources, the combination of all five tools provided the opportunity for both methodological and data triangulation, with each piece informing the other, at times clarifying or explaining inconsistencies. For example, when Richard's beliefs survey indicated that he primarily held beliefs of a constructivist nature, observing Richard's actual teaching practice contradicted these findings. The interview tool allowed for further clarification. When delving further into Richard's belief system, Richard admitted that although he stated beliefs typically classified as constructivist, he felt strongly that traditional teaching methods were best.

The factor analysis completed in Part I of the study provided a lens through which to understand the performance of each teacher on each of the specific types of items. Prior to factor analysis, several items classified as SCK ultimately factored as CCK. Without this distinction, I would have incorrectly concluded that Richard performed reasonably well on SCK items.

My own tendencies as a teacher fall into the constructivist realm and I value SCK immensely; hence, I recognize my own tendencies to appreciate classrooms where SCK is rich and a constructivist approach is being used. Ultimately, I can make no claims on the actual effects of the classroom experiences I observed on student experiences and learning.

Future Questions and Challenges

Although this study is limited in many ways, it is apparent that a large number of students in each of the ten teachers' classrooms likely experience mathematics differently in the classroom. There is a clear problem of equity that results. Major findings of this study are that teachers would be more likely to provide quality mathematics instruction if:

- Their teacher preparation focused on integrated CCK, SCK and methods of teaching mathematics at the middle and secondary level
- Their teacher preparation included the opportunity for exposure to methods courses that helped teachers understand quality mathematics instructional practices
- Their teacher preparation included continued exploration of beliefs and the development of a philosophy of teaching that supports quality mathematics instructional practices
- They had middle and secondary teaching experience prior to the first year of teaching where the use of and acquiring of new SCK and the exploration of beliefs could continue
- They received more support from the school in implementing practices that support high quality mathematics instruction (including the support to continue professional development to support such practices)

Each instance of specialized content knowledge use in the classroom occurred along with the use of core content knowledge (CCK) in each teaching episode recorded, strongly indicating the challenges of analyzing differences between tasks that represent CCK and SCK. This challenge was experienced by Hill (2007). However, this task is vital because although CCK is important, SCK enhances students exposure to the actual building blocks of knowledge.

Current policy initiatives aim at providing a high quality experience in learning mathematics by placing in every classroom a highly qualified teacher, equipped with a full teacher certification, a bachelor's degree and demonstrate competence in the subject of mathematics knowledge and teaching. This set of criteria is very limiting; it provides only a small picture of a teacher's experience relevant to his or her teaching quality. Within this study's small sample, there is a significant amount of variance in teacher credentials, teachers' beliefs, SCK knowledge level, teaching practice and ultimately, serious equity concerns tied to classroom instruction.

Every teacher in this study holds some level of common content knowledge in mathematics but significant difference in their use of specialized content knowledge ultimately played a major role in quality mathematics instruction. Teachers without specialized content knowledge – and the ability and motivation to use that knowledge to inform and improve their practice -- are analogous to cooks resigned to following a basic recipe. We need to focus attention and resources on helping teachers hone their beliefs about teaching and learning or beliefs, and enhance their opportunities to integrate specialized content knowledge into their instruction and ongoing professional development.

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APPENDIX A: SURVEYS AND INTERVIEW GUIDES



UNIVERSITY of NEW HAMPSHIRE

The Multifaceted Nature of Mathematics Knowledge for Teaching: *Understanding the Use of Teachers' Specialized Content Knowledge and the Role of Teachers' Beliefs from a Practice-based Perspective*

PART I

Introduction and Aims

Algebra as a gateway course. There is a well-documented importance of middle school exposure to algebraic topics on high school students' mathematics achievement. "Even when controlling for prior achievement, course-taking patterns in middle school mathematics resulted in significant inequities in later course-taking in grades 11 and 12" (Wang & Goldschmidt, 2003). The role of teacher content knowledge in effective teaching cannot be understated. However, much research in the areas of teacher content knowledge has been focused on the areas of number sense at the elementary grade level. "Fewer efforts have focused on teachers' knowledge of student thinking about algebraic ideas in middle grades – a period that marks a significant transition from the concrete arithmetic reasoning of elementary school mathematics to the increasingly complex, algebraic reasoning required for high school mathematics and beyond" (Asquith, et. Al, 2007, p. 251).

The aim of this particular study is to investigate specialized teacher content knowledge, knowledge use *in teaching*. Multiple forms of data will be collected to investigate this type of knowledge. **This document pertains to part I only of the study.**

Research Protocol

I will be using a secondary/existing dataset received from Dr. Heather Hill, Associate Professor of Education, Harvard Graduate School of Education, Gutman 445, Appian Way, Cambridge, Massachusetts, 02138, heather_hill@gse.harvard.edu, Phone 617-495-1898. See Appendix I for permission given to use the dataset for my dissertation (permission given by e-mail from Heather Hill).

Data Collection

The MKT dataset resulted from a larger project (Learning Mathematics for Teaching or LMT) that initially intended to develop a measurement tool to uncover, understand and measure teacher content knowledge in mathematics. In gathering study participants for this project, schools were selected first, from the National Center for Education Statistics (NCES) Common Core Database (CCD) for the school year 2002-2003. This database includes information gathered from all of the

United States' public schools. **The CCD is the result of annual** collection fiscal of data about all public schools and districts in the United States. The data are supplied by state education agency officials and include information that describes schools and school districts, including name, address, and phone number; descriptive information about students and staff and other demographics (NCES, 2011). According to Hill (2007), the 1202 middle schools chosen were housed in the CCD Preliminary School Universe file. The 1202 middle schools were then stratified first by region and urbanicity, then by probability proportional to the middle school's size. Since the structure of middle schools varies across the United States, Hill chose to define middle schools as those who had at least 10 students in grades 6-8. That is, data from schools with grades 5 or 9 were omitted (Hill, 2007).

Hill (2007) goes on to explain that after schools were chosen, teacher lists were obtained from Quality Educational Data (QED), a database which houses teacher lists for many U. S. public schools. This accounted for 75% of the teachers resulting from the chosen schools. With some teacher lists inaccessible through the QED database, the selection of the other 25% was attempted by the University of Michigan Institute for Social Research (ISR) by contacting the remaining schools directly. The end result was 1065 schools, of which 1000 were chosen. For each of these schools, a teacher was chosen at random from middle school grades 6-8 (Hill, 2007).

Teachers responded to the survey via mail with a 64% response rate. It is important to note that teachers received a fifty dollar incentive to complete the survey (Hill, 2007).

My experience with the Research Paradigm

My experience with quantitative analysis.

I have recently participated in training with the MQI Working Group at the Harvard School of Education to use the above stated tools used to evaluate the use of mathematical content knowledge in teaching. As a result of this training, I have reviewed research in the area of pedagogical content knowledge in mathematics and a history and content of the framework that led to the creation of the tool. The initial starting point of this review was in the year 2005, when I entered graduate school and have since completed the M.S.T. Mathematics program here at UNH, with a substantial focus on mathematics content knowledge for teaching as described in Deborah Ball's work cited below.

Use of Data

Data will be stored in my office, Morrill 1 in a locked cabinet. Data will be used for my dissertation in the Department of Education. Data will also be used for quantitative analysis and possibly be presented or published in a forum of peers outside of the University of New Hampshire. For this purpose, no identifying information will be used for participants. My faculty advisors, Dr. Thomas H. Schram, will have access to this data. At the end of the study, the recordings will be kept for future research in a locked cabinet.

I will be collecting the interview and observation data.

Consent

Consent is not necessary because the data have already been collected.

Risks

This study presents no more than minimal risk to participants.

Benefits

There are no direct benefits for participants; however, knowledge gathered will be used to further understand the content knowledge base necessary for teaching mathematics.

References

- Alabali, M.W., Asquith, P., Knuth, E. J., & Stephens, A.C. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning*, 9(3), 249-272.
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The Multifaceted Nature of Mathematics Knowledge for Teaching:
Understanding the Use of Teachers' Specialized Content Knowledge
and
the Role of Teachers' Beliefs from a Practice-based Perspective

PART II

Introduction and Aims

Algebra is a gateway course. There is a well-documented importance of middle school exposure to algebraic topics on high school students' mathematics achievement. "Even when controlling for prior achievement, course-taking patterns in middle school mathematics resulted in significant inequities in later course-taking in grades 11 and 12" (Wang & Goldschmidt, 2003). The role of teacher content knowledge in effective teaching cannot be understated. However, much research in the area of teacher content knowledge has been focused on the areas of number sense at the elementary grade level. "Fewer efforts have focused on teachers' knowledge of student thinking about algebraic ideas in middle grades – a period that marks a significant transition from the concrete arithmetic reasoning of elementary school mathematics to the increasingly complex, algebraic reasoning required for high school mathematics and beyond" (Asquith, et. Al, 2007, p. 251).

The aim of this particular study is to investigate specialized teacher content knowledge at the middle school level with a focus on Algebra. However, specialized content knowledge use is only one type of knowledge necessary for quality mathematics teaching. Another form of teachers' knowledge, teachers' beliefs, has been regarded for decades as central knowledge used in mathematics teaching (Anderson & Piazza, 1996; Battista, 1994; Cooney & Shealy, 1997; Ernest, 1989; Yadav & Koehler, 2004). For example, teachers who believe students construct their own knowledge during instruction generally make significantly different choices in their uses of representations and other pedagogical tools that then impact student learning in major ways. Hence, I chose to investigate the role of teachers' beliefs in specialized content knowledge use. Multiple forms of data will be collected to investigate both types of teacher knowledge (noting teachers' beliefs as teachers' knowledge). **This document pertains to Part II of the research study only. (Part II does contain a five-part study – see below).**

Research Protocol

Ten teachers will selected based on being a middle school mathematics teacher in New Hampshire, Massachusetts or Maine, currently teaching Algebra. Contact information for the teachers will be obtained on-line at the school website by the researcher and the teachers will be sent an e-mail by the researcher with the study description and participation letter (Educator Consent Document).

If teachers choose to participate, Part I of the study will involve teachers taking a voluntary 45-minute on-line multiple choice self-report survey, the Mathematical Knowledge for Teaching (MKT) Instrument developed by the Learning Mathematics for Teaching Project (2008 MKT Released

Items and Solutions). The survey data will be collected by Survey Monkey, however, there will be link to the online TAKS system, which is actually collected by and housed at the UM Institute for Social Research (Teachers will receive a letter to help with the log-in process, see Appendix 3). However, only the researcher will have access to this data. Security information regarding the Survey Monkey site is provided at http://www.surveymonkey.com/Monkey_Security.aspx. Security information regarding the TKAS system link can be accessed through Katherine Mikesell with TKAS, Katherine Mikesell (kmikesel@isr.umich.edu). Per her e-mail, which is attached, "Although the content and research component of TKAS is based at the University of Michigan School of Education, the assessment information and survey data for this study is actually collected by and housed at the UM Institute for Social Research, which is the world's largest academic social science research organization. Because the institute conducts so many survey-based studies involving sensitive information (adolescent drug use, suicide in the military, income dynamics) state of the art data security measures in network security, firewall systems, and data encryption are crucial."

In part II, teachers will participate in a paper-based, voluntary beliefs survey that will last approximately one-half hour focused on understanding teachers' beliefs about mathematics instruction and mathematics student learning. (Beliefs Survey attached). In part III, teachers will then participate in a voluntary structured interview developed by the researcher (Structured Interview). In the structured interview, participants will talk-aloud through mathematics teaching tasks included in the document Structured Interview. This interview will be video-taped. In part IV, teachers will then participate in a voluntary focused interview developed by the researcher (Focused Interview) where participants will watch the video from Part III and provide retrospective comments and educational history information. This interview will be video-taped as well. In Part V of the study, the researcher will then observe the teacher in the classroom and the classroom observation will be audio-taped; observations will occur on three to five occasions over a full class period each within a three-week period. Field notes will be taken with the goal of investigating how specialized content knowledge is used in the classroom, as well as the role of teachers' beliefs. During observations, I will be looking at the following instructional activities and investigating the quality of instruction in the following areas:

1. Working with students and mathematics
2. Richness of the mathematics
3. Error and imprecision
4. Classroom work is connected to mathematics

I am looking at the quality of teacher actions and teacher responses only, in these areas. For example, if a teacher gives an explanation about fractions that includes an error, this error would be noted in the field notes. Notes will be analyzed qualitatively for themes in the above four areas upon the completion of the study. Further information on these categories can be found in Appendix 6.

The MKT and beliefs survey, observations and interviews will take place at the school site at the participant's convenience. Participants may withdraw at any time from participating in the study, but participants must complete all study activities described above and in the consent form to receive a \$50.00 compensation at the conclusion of the study (the day of the last observation).

My experience with the Research Paradigm

I have recently participated in training with the MQI Working Group at the Harvard School of Education to use the above stated tools used to evaluate the use of mathematical content knowledge in teaching. As a result of this training, I have reviewed research in the area of pedagogical content knowledge in mathematics and a history and content of the framework that led to the creation of the tool. The initial starting point of this review was in the year 2005, when I entered graduate school and have since completed the M.S.T. Mathematics program here at UNH, with a substantial focus on mathematics content knowledge for teaching as described in Deborah Ball's work cited below.

Use of Data

Data will be stored in my office, Morrill 1 in a locked cabinet. Data will be used for my dissertation in the Department of Education. Data will also be used for both quantitative and qualitative analysis and possibly be presented or published in a forum of peers outside of the University of New Hampshire. For this purpose, no identifying information will be used for participants. Dr. Nodie Oja, will have access to this data. At the end of the study, the recordings will be kept for future research in a locked cabinet.

I will be collecting the survey, interview and observation data.

Risks

This study presents no more than minimal risk to participants.

Benefits

There are no direct benefits for participants; however, knowledge gathered will be used to further understand the content knowledge base necessary for teaching mathematics.

References

- Alabali, M.W., Asquith, P., Knuth, E. J., & Stephens, A.C. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning*, 9(3), 249-272.
- Ball, D. L., Hill, H. C. & Schilling, S. G. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, 105(1), 11-30.
- Wang, J., & Goldschmidt, P. (2003). Importance of Middle School Mathematics on High School Students' Mathematics Achievement. *The Journal of Educational Research*. 97(1), 3-17.

The Multifaceted Nature of Mathematics Knowledge for Teaching:

Understanding the Use of Teachers' Specialized Content Knowledge

and

the Role of Teachers' Beliefs from a Practice-based Perspective

PART II

Approach to Teaching Math Survey (Staub & Stern, 2002)¹

Directions: In the following pages, you'll find a variety of statements. For each statement, decide to what degree you agree with the statement. If you completely agree with the statement, circle A. If you agree with the statement somewhat, circle B. If you somewhat disagree with the statement, circle D. If you totally disagree with the statement, circle E. **If you aren't sure if you understand the statement or are undecided about the statement, circle C.**

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

Think carefully about and process each statement without spending too much time on any one statement. Work expediently and carefully.

There are no right or wrong answers. The best answer is the accurate reporting of your personal opinion about each statement. It's important to answer each statement according to your own personal outlook. This survey will be used for research purposes.

The two terms below, which will be used often, should be explained.

Computational Procedure: This term refers to concrete approaches or strategies for the execution of computation procedures.

Knowledge of Number Facts: The ability to recall, from memory, specific mathematical operations (i.e. $8+8=16$) without having to do the computation.

Please fill out the survey below.

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

1. Students should already have a textbook with math facts before they have mastered computation, algorithms or procedures.

A B C D E

2. Teachers should encourage students to find their own ways of solving mathematical problems even when it is an inefficient use of time.

A B C D E

3. Students should understand procedures before just using them.

A B C D E

4. During lessons, students should already receive simple word problems before too much time is spent practicing math skills.

A B C D E

5. Teachers should share detailed approaches to solving word problems.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

6. Students should understand math procedures before they memorize math facts.

A B C D E

7. Teachers should show how simple it is to solve word problem before students solve word problems on their own.

A B C D E

8. Attention to key-words, such as “more”, is a big help for students when they are solving word problems.

A B C D E

9. In school, math should be taught in such a way that students can discover their own connections.

A B C D E

10. Even students, who do not yet have a solid knowledge of math facts, can successfully solve problems.

A B C D E

11. In order to learn math, a student must be able to listen well.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

12. Most middle school students can solve simple word problems on their own.

A B C D E

13. Students should already have solved many simple word problems before it is expected that they memorize math facts or use formulas/procedures.

A B C D E

14. Students should be able to solve a problem the way in which it was taught in a lesson.

A B C D E

15. Most middle school students must be shown how easy it is to solve word problems.

A B C D E

16. A student's true mathematics ability level is shown when he/she solves a math problem in writing.

A B C D E

17. Students learn to solve word problems better when one method is concentrated on instead of learning many different methods at once.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

18. It is better to confront students with a wide variety of word problems.

A B C D E

19. Students learn math best when they solve word problems on their own.

A B C D E

20. Students can normally find out how easy it is to solve word problems on their own.

A B C D E

21. The acquisition of math facts should come before the comprehension of computation.

A B C D E

22. Students don't understand computation until they understand math facts.

A B C D E

23. Most students can't discover math on their own and need explicit instruction.

A B C D E

24. Before students master the execution of procedures, they should understand it.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

25. Students learn math best when they follow the teacher's instructions.

A B C D E

26. It is important for a child to discover how simple word problems are to solve on their own.

A B C D E

27. Students should be allowed to devise solutions to simple word problems before the teacher demonstrates the solution.

A B C D E

28. Students cannot be expected to understand procedures before it has been practiced.

A B C D E

29. In math, the teaching goal would be best achieved if the students found his/her own method to solve the problem.

A B C D E

30. Teachers should allow students who are having difficulties solving word problems to continue their own problem solving techniques.

A B C D E

31. Students can find solutions to math problems without instruction.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

32. It is imperative to practice math skills frequently in order to acquire math skills.

A

B

C

D

E

33. Most middle school students can find solutions to many math problems without help from adults.

A

B

C

D

E

34. Teacher should admit that students discover their own way to solve simple math problems.

A

B

C

D

E

35. It is better for students not to mix different methods when for solving word problems. Rather they should be addressed one after another.

A

B

C

D

E

36. Students should receive their first simple word problems if they have mastered a portion of their math facts/procedures.

A

B

C

D

E

37. A student's explanation of their problem solving, exposes their knowledge about math.

A

B

C

D

E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

38. With the help of appropriate materials, students develop computation skills of their own.

A B C D E

39. Before time is spent on solving a word problem, students should practice the underlying math skill.

A B C D E

40. Teachers should encourage students to solve simple word problems on their own.

A B C D E

41. Students will become good problem solvers if they can follow the instruction of the teacher.

A B C D E

42. To be successful in math, a student must be a good listener.

A B C D E

43. Students need detailed instruction in order to solve word problems.

A B C D E

A=Totally Agree

B=Somewhat Agree

C=Undecided

D=Somewhat Disagree

E=Totally Disagree

44. Students cannot be expected to understand the procedures until they master the execution of the procedures.

A B C D E

45. Students learn math best from the teacher's presentation and application.

A B C D E

¹This survey was received by e-mail from Prof. Dr. Elsbeth Stern, ETH Zürich, Institut für Verhaltenswissenschaften, UNO C 11, Universitätstrasse 41, 8092 Zürich, Switzerland, elsbeth.stern@ifv.gess.ethz.ch. The survey was received in German and translated by a professional translator, Linda Hackett, Sparhawk School, 259 Elm Street, Amesbury, MA 01913.

The Multifaceted Nature of Mathematics Knowledge for Teaching:

Understanding the Use of Teachers' Specialized Content Knowledge

and

the Role of Teachers' Beliefs from a Practice-based Perspective

PART II

Focused Interview

Directions read to participant: We are going to watch the videotape of your first interview, where you completed and talked-aloud through a variety of teaching tasks. Before we watch the videotape, we would like you to look at each of the written tasks and recall any thinking, decision-making or other information regarding what you were thinking during the task the first time you completed it.

Now, again, we are interested in what you are thinking as you watch each task. In order to do this, we will ask you to *talk aloud* as watch when you feel you can. I am able to pause the video if need be. This means that I would like you to say out loud *everything* you might say to yourself or what you are thinking. You could consider acting as if you were alone in a room watching the video by yourself. If you are silent for any length of time, I might remind you to keep talking aloud. Do you understand what I want you to do? Any questions?

I am interested in what you actually can remember rather than *what you think you might have thought*. If possible, I would like you to tell me your memory in the order in which they happened while working on the task. I would also like you to tell me if you are uncertain about a memory. No need to solve the problem again, just what you remember now from solving the problem.

Other possible retrospective interview prompts:

1. **Comprehension testing:** Can you repeat the question in your own words? What do you mean by "X"? Why do you believe "X"?
2. **Confidence testing:** How sure are you of your answer?
3. **Recall testing:** How do you remember this?
4. **Explanations:** How did you arrive at that answer?
5. **Keep talking probes:** Can you expand on this? Can you think of any other information to share?

Additional history questions:

- 1. How did you come to teach mathematics?**
- 2. What is your educational background? (Certification, Bachelors, etc.) Major field of study? Minor field of study?**
- 3. How many mathematics classes have you had? How many mathematics method courses have you taken?**
- 4. How many years of teaching and at what levels/classes?**
- 5. What is mathematic knowledge used in teaching?**
- 6. How did you own mathematics classroom experiences influence your own teaching or beliefs about mathematics?**
- 7. What else influences how you choose to teach mathematics?**
- 8. How often do you participate in mathematics professional development with some focus on content?**
- 9. How would you describe your beliefs about mathematics teaching?**
- 10. How do students best learn mathematics?**

--- On Mon, 2/14/11, Heather Hill <hillhe@gse.harvard.edu> wrote:

From: Heather Hill <hillhe@gse.harvard.edu>
Subject: Re: hello again
To: "Lauren Elizabeth" <laureneliz2@yahoo.com>
Date: Monday, February 14, 2011, 6:21 PM

Hi Lauren,

Sorry -- your email did get lost in the in-basket.

First, you do have permission to use the data.

On the dissertation front, it's not likely that I can be very helpful. I'm on sabbatical all of next year, and am really cutting down on my advising/research responsibilities in order to get some other stuff done.

Good luck!

Heather

University of New Hampshire

Research Integrity Services, Service Building
51 College Road, Durham, NH 03824-3585
Fax: 603-862-3564

08-Nov-2010

Provost, Lauren
Morrill Hall, Education Dept.
65 Whitehall Road
Amesbury, MA 01913

IRB #: 5005

Study: Study of Middle School Mathematics Teachers' Specialized Content Knowledge

Approval Date: 05-Nov-2010

The Institutional Review Board for the Protection of Human Subjects in Research (IRB) has reviewed and approved the protocol for your study as Expedited as described in Title 45, Code of Federal Regulations (CFR), Part 46, Subsection 110 with the following comment(s):

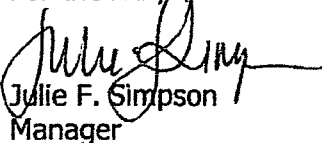
Before starting the study in a site, the researcher needs to forward to the IRB for the file a letter from the principal in support of the study, and receive approval from the IRB to start the study in that site.

Approval is granted to conduct your study as described in your protocol for one year from the approval date above. At the end of the approval date you will be asked to submit a report with regard to the involvement of human subjects in this study. If your study is still active, you may request an extension of IRB approval.

Researchers who conduct studies involving human subjects have responsibilities as outlined in the attached document, *Responsibilities of Directors of Research Studies Involving Human Subjects*. (This document is also available at <http://www.unh.edu/osr/compliance/irb.html>.) Please read this document carefully before commencing your work involving human subjects.

If you have questions or concerns about your study or this approval, please feel free to contact me at 603-862-2003 or Julie.simpson@unh.edu. Please refer to the IRB # above in all correspondence related to this study. The IRB wishes you success with your research.

For the IRB,


Julie F. Simpson
Manager

cc: File
Oja, Sharon